ARTICLE

The Network Evolution of the Internationalizing Firm

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ABSTRACT
This paper analyses the network evolution of the internationalizing firm, focusing on a generative mechanism called preferential attachment. Preferential attachment means that more connected actors are likely to form more relationships. This paper uses longitudinal quantitative data of a single case of the internationalization of an American multinational firm’s operational division in the Brazilian market. The data analysis is based on Clauset et al.’s (2009) computational algorithm and PAFit, a new statistical method. The aim is to identify the extent to which the network evolution follows a power-law distribution and the degree to which preferential attachment affects the network evolution. It finds that the network evolution of the internationalizing firm follows a power-law distribution. It is affected by a sub-linear form of preferential attachment. Few actors accumulate a disproportionately high number of relationships. The preferential attachment does not homogeneously manifest itself in the network evolution. It has a strong effect on the network onset. This paper contributes by advancing a relational, process-based approach to the internationalization of the firm. It shows that the network evolution of the internationalizing firm grows over time and becomes sparser. More connected actors form hubs, meaning increased status, more power and resources.

KEYWORDS
Internationalization, Network Evolution, Power-law Distribution, Preferential Attachment

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1. INTRODUCTION

Scholarly work usually suggests that the network of the internationalizing firm is not static (e.g., Vahlne & Johanson, 2020). It is dynamic as it changes with time about compositional actors and structural properties (Coviello, 2006; Lamb, Sandberg, & Liesch, 2011). Although this idea has taken on almost a truism, surprisingly, research on the internationalization of the firm relies heavily on comparative statics (Montoro-Sanchez, Diez-Vial, & Belso-Martínez, 2018).

This practice is problematic as research implicitly assumes that the network of the internationalizing firm is invariant to time. However, “[…] network relationships are process-based, and process is, by definition, dynamic rather than static” (Coviello, 2006, p.715). As a result, it is still unclear how relationships are formed and terminated to establish and sustain foreign operations (Hohenthal, Johanson, & Johanson, 2014). There is also a lack of understanding about the generative mechanisms that drive the network evolution of the internationalizing firm (Cuypers, Ertug, Cantwell, Zaheer, & Kilduff, 2020). In this sense, what is missing in the literature is not what networks are but how and why they come into being (Vahlne & Johanson, 2017, 2020; Kurt & Kurt, 2020).

This paper helps correct this deficiency by analyzing the network evolution of the internationalizing firm. In doing so, it focuses on preferential attachment as a generative mechanism of the network evolution (Barabási & Albert, 1999). In networks in which preferential attachment is a dominant mechanism, a few actors accumulate relationships faster than most actors. As a result, they become not only more resourceful and prestigious, but they also occupy central positions in the networks (Barabási, 2016). This “rich getting forever rich” dynamic (Merton, 1968, p. 610) explains the origins of hubs and peripheral actors in networks (Andriani & McKelvey, 2009; Kurt & Kurt, 2020).

Against this backdrop, this paper addresses two research questions: how do networks in which the internationalizing firm is embedded evolve? To what extent does preferential attachment explain the evolution of this network? To answer these questions, the paper’s empirical analysis relies on a quantitative case of the internationalization process of a multinational firm, herein OilTubs (fictitious name), in the Brazilian market.

This paper adds to knowledge by advancing a relational, process-based approach to the internationalization of the firm. It shows that the internationalizing firm follows a power-law distribution. It also demonstrates that, over time, some actors accumulate a disproportionally high number of relationships. That means that preferential attachment affects the network evolution of the internationalizing firm. Another novelty of the paper is that it finds evidence that preferential attachment assumes a sub-linear form. Due to costs, communication and managerial limitations, as well as node age, relationship accumulation decays over time. Finally, this paper responds to a seemingly neglected call for further research on this topic (e.g., Hohenthal et al., 2014; Cuypers et al., 2020; Kurt & Kurt, 2020).

2. THEORETICAL BACKGROUND

2.1. NETWORKS IN THE INTERNATIONALIZATION OF THE FIRM

It has been long acknowledged that the internationalization of the firm is not the outcome of a solitary firm’s resources and capabilities (Hult, Gonzalez-Perez, & Lagerstrom, 2020; Johanson & Valhne, 2009; Vahlne & Johanson, 2017, 2020, 2021). To a greater or lesser extent, the internationalizing firm is embedded in networks of relationships (Coviello, 2006; Johanson &
Within these networks, the firm recurrently transacts tangible (e.g., raw materials) and intangible (e.g., referrals) resources with several actors, such as buyers and suppliers (Johanson & Mattsson, 1988; Vahlne & Johanson, 2021).

This relational view of the internationalization of the firm has attracted attention for the past twenty years (Hult et al., 2020; Vahlne & Johanson, 2020, 2021). On the one hand, most of the studies assume that the network precedes the internationalization of the firm (Hohenthal et al., 2014). First, the networks are repositories of new information, international opportunities, and complementary resources and capabilities (Chandra, Styles, & Wilkinson, 2009). Second, Born-global and new venture firms accelerate internationalization by simultaneously forming networks in various foreign markets (Coviello, 2006). Third, domestic networks are bridges to cross-border networks (Montoro-Sanchez et al., 2018). Fourth, firms embedded in networks can share knowledge and experience, thus decreasing internationalization risks (Figueira-de-Lemos, Johanson, & Vahlne, 2011).

On the other hand, fewer studies explain the emergence of networks in the internationalization of the firm. They assume that the networks are consequences of internationalization (Hohenthal et al., 2014). For example, Senik, Scott-Ladd, Entrekin, and Adham (2011) suggest that the internationalizing firm forms networks because it participates in international fairs and roadshows. Ojala (2009) posits that the internationalizing firm proactively seeks international opportunities, thus paving the way for network formation.

Despite such differences, these studies neglect how the network emerges and evolves in the internationalization of the firm. They also pay little attention to how generative mechanisms affect this evolutionary process. These omissions are paradoxical because most of the studies are embryonically rooted in the Uppsala model that strongly emphasizes processes (Johanson & Vahlne, 1977; Welch et al., 2016; Vahlne & Johanson, 2020, 2021).

A handful of studies depart from this tradition. For example, Coviello and Munro (1997) found that networks not only drove the entry of small software firms into foreign markets, but also spurred product development and diversification. Coviello (2006) showed that the network of international new ventures became larger but sparser with time. Moraes, Rocha, and Silva (2017) unveiled two generic processes of network evolution. However, these findings come from research on the dyadic relationships of the firm. That may compromise the understanding of the network evolution of the internationalizing firm twofold. First, it is still debatable whether network subset results are valid to the internationalizing firm’s whole network. Second, it is also an open question of whether a generative mechanism operating at a dyadic level also operates at a network level (Gay, 2012).

In a nutshell, International Business (IB) has paid little attention to network change in the internationalization of the firm. As a result, knowledge of the network evolution of the internationalizing firm and generative mechanisms remains restricted. Per Cuypers, Ertug, Cantwell, Zaheer, and Kilduff (2020), a fruitful avenue to address these shortcomings is to use the statistical methods developed to study complex networks (e.g., Barabási & Albert, 1999). By appropriating these studies, IB may overcome limitations about the understanding of the network evolution of the internationalizing firm (Kurt & Kurt, 2020).

### 2.2. Network Evolution and Preferential Attachment

This paper proposes that the network evolution of the internationalizing firm has its roots in the relationships formed between the firm and its counterparts (Eriksson, 2016). First, the internationalization process can be decomposed into events that unfold over time (Welch et al.,
These events relate to each other in that an event paves the way for the next event (Eriksson, 2016). Scholars usually map out events in the internationalization of the firm as operational mode changes used by the firm to serve a particular foreign market (Benito, Petersen, & Welch, 2009).

Second, the internationalizing firm and its counterparts may form or terminate relationships in each event. Forming relationships happens between a new actor and an actor already embedded in the network; among actors that are new to the network; or among disconnected but embedded actors in the network. Terminating relationships implies either interrupting transactions between two actors or exiting the network (Ghoshal, Chi, & Barabási, 2013).

This way of conceptualizing the network evolution of the internationalizing firm implies that the network is an open system. Actors enter and exit as it evolves. Such an understanding is close to Barabási and Albert’s (1999) model of network evolution named Scale-Free Network (SF). Accordingly, actors do not enter and exit the network at random. Few actors attract more relationships, resulting in networks that depart from the Gaussian distribution but approximate the power-law distribution. This distribution has unstable means, infinite variance, and displays fat tails caused by hubs, e.g., highly connected actors (Andriani & McKelvey, 2009). This network has also been called Core-Periphery (Sun & Liu, 2016; Kurt & Kurt, 2020).

Although entering and exiting networks do not necessarily occur serendipitously, small differences or random events at the beginning of the network evolution may amplify, yielding massive differences with time (Aguinis, O’Boyle, Gonzalez-Mulé, & Joo, 2016). In this sense, the network evolution of the internationalizing firm is path-dependent (Araujo & Rezende, 2003). The current structure of the network closes future options. Hence, it restricts the paths that the network evolution may take.

The evolution of SF networks is shaped by the preferential attachment mechanism (Barabási & Albert, 1999). Preferential attachment means that the “amount of some quantity distributed among the members of a population increases with the amount of the quantity they already possess” (Pham, Sheridan, & Shimodaira, 2016, p.2). That is, actors with higher degrees tend to increase their degree faster than do actors with lower degrees. In the simplest form, preferential attachment is a generative mechanism of the network evolution that points to a disproportional accumulation of relationships by few actors.

Highly connected actors enjoy social and economic advantages stemming from their high degree (Lazega & Jourda, 2015). First, they may be preferred for transactions based on quality judgments, even though quality cannot be guaranteed ex-ante. It is generally assumed that highly connected actors have positive attributes because they are well connected (Barabási, 2016). Second, highly connected actors have a higher status in the networks. As status is usually associated with performance, their survival chances are higher (Sauder, Lynn, & Podolny, 2012). Third, such actors have more access to information and knowledge because such resources usually flow through them (Ahuja et al., 2012). Fourth, highly connected actors have more power in the networks, permitting them, for example, to discriminate prices. These social and economic advantages are self-reinforcing. The more the actors become resourceful, the more they attract more relationships. The more they form new relationships, the more they control critical resources in the networks (Dahlander & McFarland, 2013).

In its original form, preferential attachment is a linear generative mechanism of network evolution. The accumulation of relationships by actors is proportional to their degree. However, this may not necessarily hold true in social networks (Broido & Clauset, 2018). In these networks, factors such as costs, managerial capabilities, and node age limit actors to accumulate relationships proportional to their degree. Specifically, forming relationships are costly. There are the costs of
searching, contracting, and monitoring potential partners (Williamson, 1985). Likewise, managing a portfolio of relationships is not cost-free (Dagnino, Levanti, & Destri, 2016). Partners usually need some dedication and attention to be interested in maintaining relationships. Given that, it is not trivial to reconcile conflicting interests and requests (Dahland & McFarland, 2013). Yet, actors age (Gay, 2012). As a result, they face difficulties in forming new relationships. They may also exit the network. Lastly, hubs constituted by highly connected actors may become obsolete concerning innovation because knowledge becomes homogeneous (Gulati et al., 2012).

Therefore, it is an open question of whether SF networks exist in their purest form in social networks (Broido & Clauset, 2018). It is still unclear whether preferential attachment assumes a linear function (Gay, 2012). As aforementioned, this lack of consensus is more evident in IB (Hohenthal et al., 2014; Cuypers et al., 2020; Kurt & Kurt, 2020), thus constituting a fertile ground to advance a relational, process-based approach to the network evolution of the internationalizing firm.

3. RESEARCH DESIGN

3.1. CASE SELECTION

We designed a research protocol to guide the case selection. We established that the empirical case should be a manufacturing firm because research on the internationalization of the firm from a relational standpoint relies mostly on this firm type (Welch et al., 2016; Vahlne & Johanson, 2020). To analyze the network evolution, we considered that it should unfold during a “long” period. To embrace events that precede the market entry (e.g., time to internationalize) and post-entry events, we also established that the firm should be involved with mode entry and change. Finally, as the network evolution is a relational phenomenon (Coviello, 2006), we deemed that data on actors and counterparts were mandatory, implying collecting secondary and primary data from and about several actors.

Because the collection of longitudinal data covering a “long” period is demanding, it is common to select cases based on data accessibility (Langley, 1999). Thus, due to previous relationships, one of the authors approached OilTubs. At that moment, the aim was to collect data and carry out a preliminary analysis to decide about additional cases. Surprisingly, we have been granted rare access to key individuals worldwide and managed to collect a vast amount of secondary and primary data. In the end, we realized that this case was enough to produce a relevant piece of research.

3.2. DATA SOURCES

The research protocol also indicated how to collect secondary and primary data. Initially, we built an interview schedule to cover the main topics: the process of the internationalization of the firm, the relationships the firm developed to progress with internationalization, and the actors involved in such process. Then, we established that we should collect data from most of the actors mentioned in the materials. For example, if the annual report cited a supplier, we requested an interview. Likewise, if an interviewee commented on a particular buyer, we would seek data about it and ask the interviewee to introduce us to it. This snowballing process would take place until we reached data saturation.
Given that, we collected secondary and primary data. Table 1 shows the sources of secondary data, totaling 973 double-spaced pages of documents.

<table>
<thead>
<tr>
<th>#</th>
<th>Source</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Annual reports for the period from 2002 to 2012</td>
<td>505</td>
</tr>
<tr>
<td>2</td>
<td>Marketing Materials</td>
<td>72</td>
</tr>
<tr>
<td>3</td>
<td>Contracts and written records of negotiations between OilTubs and clients, suppliers, and government organizations</td>
<td>148</td>
</tr>
<tr>
<td>4</td>
<td>Major customer strategic plans</td>
<td>93</td>
</tr>
<tr>
<td>5</td>
<td>Investment plans and manufacturing plant building plans</td>
<td>95</td>
</tr>
<tr>
<td>6</td>
<td>News about the firm and the oil gas industry published in newspapers and specialized magazines</td>
<td>60</td>
</tr>
<tr>
<td>7</td>
<td>Websites of the firm, buyers, and suppliers</td>
<td>–</td>
</tr>
</tbody>
</table>

*Source: Authors.*

We also carried out 37 interviews with individuals from the firm, its subsidiaries, the major buyers and suppliers, and the local government agents. These face-to-face interviews took place in five countries: Brazil, the USA, the Netherlands, Singapore, and Argentina. Each interview was digitally recorded and transcribed verbatim. The total recorded time is 591 minutes (43 minutes per interview on average). We returned each transcribed interview to the respective interviewees for checking accuracy and complementing missing data. Due to space limits, the interview table is unreported in this paper. However, it is available from the first author upon request.

Based on the secondary data and informal conversations with key individuals, before we initiated the interviews, we identified the critical events that could illustrate the internationalization process of OilTubs in the Brazilian market. According to Eriksson (2016), critical events are empirical facts registered on the firm’s internal documents and annual reports. They are also retrieved from individuals’ memories. In this sense, the act of remembering means that individuals talk about events stored in their conscious minds, i.e., events relevant to them.

Thus, we initiated the interviews by asking the individuals to talk about the internationalization process of OilTubs in the Brazilian market. Not all individuals had full knowledge of the entire process. Hence, they were encouraged to give details about the events they knew most. Next, we presented a figure showing the internationalization process of OilTubs and asked for critical comments. Interestingly, they added no event, which means that the figure was an accurate visual portrait of the internationalization process of OilTubs in the Brazilian market. Finally, we asked the interviewees to identify the business and nonbusiness actors that participated in each critical event. For each nominated actor, we asked for details of her participation.

By contrasting secondary and primary data, subsequently, we crosschecked the business and nonbusiness actors’ participation in each event. We required at least two instances of evidence; one compulsorily found in the secondary data.
3.3. Analytic Strategy

We followed the systematic approach to analyze a single case study (Dubois & Gaade, 2002), which involves a continuous conversation between data and theory. After we produced several network metrics over various periods, such as network size, degree distribution, and network density (Wasserman & Faust, 1994), we found that some actors had built the majority of the relationships. This finding was consistent over the years. Thus, we revisited the literature on the generative mechanisms of network evolution (e.g., Ahuja et al., 2012), focusing on papers on the internationalization of the firm (e.g., Cuypers et al., 2020; Hohenthal et al., 2014). This led us to the literature on complex networks (e.g., Barabási & Albert, 1999). That literature has not only progressed considerably but has also developed statistical methods to analyze network evolution (e.g., Pham et al., 2015, 2016). We believed that using complex network studies, we would produce relevant research on the network evolution of the internationalizing firm.

In more detail, the empirical analysis of the paper relies on binary network data (Pham et al., 2017). For each event of the internationalization of OilTubs, we constructed a quadratic, one mode matrix indicating whether a business or nonbusiness actor participated in an event. For example, if actor A and actor B participated in the event X, we coded it as 1; otherwise, we coded it as 0. This process produced 21 matrixes containing 29,368 binary data. Three individuals independently built the matrixes. By contrasting them, we identified and corrected discrepancies.

Next, we grouped the 21 matrixes into five matrixes, each corresponding to an episode of the internationalization process of OilTubs in the Brazilian market. In this sense, the critical events (21 matrixes) were grouped into episodes (5 matrixes). While a critical event represents an empirical fact retrieved from secondary and primary data, an episode contains similar nature events (Eriksson, 2016). Therefore, the analysis of the internationalization of the firm considers not only structure (e.g., operational mode and subsidiary role) but also respective activities. For example, direct export (operational mode) involves several activities performed by the firm and its counterparts. Exporting, in this case, is the episode, whereas the activities such as identifying potential customers and contracting are critical events.

We chose the expanding window to analyze the network evolution of OilTubs. This technique refers to a cumulative view of network evolution (Doreian, 1986). Thus, later episodes are brought into the analysis successively. For example, when episode #4 is considered, a fourth matrix is produced, representing the network evolution up to that point. This matrix is the accumulation of matrixes #1, #2, and #3. To control for relationship decay, we followed standard research practices, discarding actors not mentioned in five years (Gulati et al., 2012).

Although we collected qualitative and quantitative data, the empirical analysis of this paper relies on quantitative data. In addition to space constraints, this decision was inspired by a recent IB call for “[…] invest(ing) in longitudinal data collection so as to conduct quantitative time-series analysis” (Vahlne & Johanson, 2020, p.7). In this sense, the statistical analysis identifies the extent to which the network evolution of OilTubs follows a power-law distribution. It also points out the degree to which preferential attachment affects network evolution.
To accomplish the former, we employed the computational routines proposed by Clauset, Shalizi, & Newman (2009) (see the original paper for a thorough discussion). First, Broido and Clauset (2018) argue that it is not trivial to identify pure SF networks, especially in the Social Sciences, since not all values have a perfect adjustment to power-law. Hence, a minimum value ($K_{min}$), corresponding to the distribution tail, is defined. A power-law is found for values greater than $K_{min}$. Second, the exponent of the curve is calculated ($\gamma$). SF networks have exponents in the range between 2 and 3 (Ghoshal et al., 2013). Third, the goodness-of-fit (gof) is calculated. It compares the distance between the empirical network data and the hypothesized model (null hypothesis). Fourth, the Kolgomorov-Smith test is performed to calculate the fit of the model (p-value). According to Clauset et al. (2009), the power-law distribution can be rejected if the p-value is lower than 0.10. Finally, the power-law distribution results are compared to competing distributions, such as exponential, Poisson, and log-normal. “Positive values of the log likelihood ratios indicate that the power law is favored with the alternatives” (Clauset et al., 2009, p.28).

About the latter, this paper employs a non-parametric estimation model called PAFit (Pham et al., 2015, 2016, 2017). The model is non-parametric because it does not assume a linear form of preferential attachment. Hence, the preferential attachment may assume either a sub-linear or a super-linear form. Either preferential attachment form means that the accumulation of relationships is not linearly proportional to the actors’ degree.

Specifically, the method calculates the attachment kernel ($Ak$) between two evolving networks, each corresponding to an expanding window. $Ak$ is a function that indicates the probability of an actor to form relationships based on its degree. If the results of $Ak$ assume values that are “an increasing function of average” (Pham et al., 2015, p.2), it is said that the actors that have accumulated more relationships are likely to build more relationships than the actors that have fewer ones. In other words, the closer $Ak$ (in fact, the $Ak$ log-linear) is to 1, the stronger preferential attachment affects the network evolution. In a personal communication (e-mail dated 23/11/2016), Pham points out that exponents superior to 0.5 indicate a strong preferential attachment. Due to space constraints, we refer the reader to the original papers for a thorough discussion of PAFit.

4. RESULTS

4.1. BACKGROUND

OilTubs is an operational division of an American multinational firm that controls 237 firms and operates in 48 countries by establishing sales subsidiaries, manufacturing plants, and R&D centers. It manufactures and sells composite pipes and fittings used by firms operating both onshore and offshore in the oil and gas industry.

The internationalization of OilTubs in the Brazilian market comprises five episodes covering 1996-2013 (see Table 2). The first episode, Exporting, is the beginning of operations in the Brazilian market. Initially, the firm paid some technical visits to potential clients to assess the Brazilian market potential. Soon afterward, it began exporting sporadically to Petrobras, the Brazilian state-owned petroleum firm. Concurrently, it transferred some technology to indigenous firms because knowledge transfer was mandatory to transact with Petrobras. Exporting of onshore and subsequently offshore products became continuous. The network at the onset comprised ten actors. At the end of episode #1, the network became larger (23 actors).

The second episode of the internationalization of OilTubs in the Brazilian market is named Acquisition. It comprises events that lead to the acquisition of an indigenous firm by OilTubs.
For example, OilTubs developed relationships with local suppliers. It also launches a program called Local Content to manufacture pipelines and fittings locally. To do so, OilTubs finished a joint venture in Argentina because of a potential surplus. The joint venture aimed to manufacture and export pipelines and fittings to the Brazilian market. The network size in episode #2 went from 27 actors to 23 actors.

**Table 2**

*The Internationalization of OilTubs in the Brazilian market*

<table>
<thead>
<tr>
<th>The internationalization process</th>
<th>Network size</th>
</tr>
</thead>
</table>
| **EPISODE I**  
I. Exporting (Entry into the Brazilian market)  
I-a. Technical visits | 10  
I-b. Exports of onshore products | 13  
I-c. Development of national competitor (Phase 1) | 21  
I-d. Exports of offshore products | 22  
I-e. Development of national competitor (Phase 2) | 23 |
| **EPISODE II**  
II. Local Acquisition  
II-a. Joint Venture dissolution | 27  
II-b. Local Content program | 21  
II-c. Building relationships with local suppliers | 22  
II-d. Local acquisition | 20  
II-e. Development of local suppliers | 23 |
| **EPISODE III**  
III. Greenfield  
III-a. Greenfield: onshore and offshore products | 23  
III-b. Certification of products and manufacturing plant | 22  
III-c. Sales and installation of the longest Latin American pipeline | 27  
III-d. Roadshow in the manufacturing plants of the major clients | 28  
III-e. Development of products and projects together with local suppliers | 29 |
| **EPISODE IV**  
IV. International Exporting  
IV-a. Sales to other subsidiaries | 32  
IV-b. A contract for supplying eight ships and 15 platforms | 34  
IV-c. Development of projects for deepwater exploration | 34  
IV-d. Building a plant for manufacturing onshore and offshore products | 35 |
| **EPISODE V**  
V. Worldwide Acquisition  
V-a. Global strategy and product range streamline | 35  
V-b. Closing of local operations | 35 |

*Source:* Authors.
Greenfield is the title of the third episode of the internationalization of OilTubs in the Brazilian market. It involves events showing a higher resource commitment to the market. Not only did the firm set a new manufacturing plant from scratch, but it also obtained international certifications for products manufactured in Brazil. In this episode, OilTubs sold and delivered the longest pipeline in Latin America, thus requiring innovative logistics solutions. Unlike episode #3, the network size increased, going from 23 actors to 29 actors.

At this point, OilTubs has increased in size. It also became more resourceful because of the sales volume increase. Due to the high quality of its products, the Brazilian subsidiary extended its role in the multinational firm’s network by exporting pipelines and fittings to sister subsidiaries (e.g., the units that commercialized floating and production storage offloading navies and petroleum platforms). That is the fourth episode of the internationalization process of OilTubs in the Brazilian market. The network became larger: from 32 to 35 actors.

The last episode is the closing of OilTubs operations. The headquarters made a global acquisition of a firm that owned an operational division that overlapped with OilTubs. It also decided to discontinue the manufacturing of onshore products globally, resulting in local sales decrease by 60%. The Brazilian subsidiary became redundant. As a result, the headquarters divested local operations, withdrawing from the Brazilian market. The network size ended up with 35 actors.

Figure 1 shows the network topology in the five episodes of the internationalization of OilTubs in the Brazilian market. Note that the network became larger in terms of the number of actors. However, it also became sparser. Interestingly, this result is similar to Coviello’s (2006) when studying the evolution of software firms’ dyadic relationships.

Figure 1. Network Topology. (a) Episode #1; (b) Episode #2; (c) Episode #3; (d) Episode #4; (e) Episode #5
Source: Authors
The five episodes of the internationalization of OilTubs in the Brazilian market show p-values of the power-law distribution greater than 0.10. Thus, the hypothesis that the network evolution of OilTubs follows a power-law distribution is not rejected. The highest p-value is found in episode #2 (p=0.63). In this episode, OilTubs acquired an indigenous firm. As a result, it became directly and indirectly connected with actors such as suppliers and buyers of the acquired firm. In other words, episode #2 points to the highest heterogeneity of actors’ degree. OilTubs and Petrobras were the highly-connected actors, i.e., they had accumulated more relationships than did the majority of the actors in the network. The p-values decrease in the subsequent episodes of the internationalization of OilTubs in the Brazilian market, going from p=0.49 (episode #3) to p=0.26 (episode #5). This decrease is explained by the fact that other actors entered the network by accumulating relationships at rates disproportional to their degree. That is the case of an affiliate of Petrobras named Transpetro.

The results of the exponent $\gamma$ further support that OilTubs’ network in the Brazilian market follows a power-law distribution. The exponent values fall within the range between 2 and 3 in all the episodes of the internationalization of OilTubs. This is the original interval suggested by Barabási and Albert (1999) to indicate that the network evolution results in power-law distributions.

To increase the robustness of the findings, Clauset et al. (2009) suggest comparing the power-law distribution results with alternative distributions. This test is critical because some distributions, such as the log-normal distribution, also reflect heterogeneous degree distributions. According to Sheridan and Onodera (2018), this is particularly valid for networks of finite sizes, such as ours.

Table 3 shows that the p-values are only statistically significant for the Poisson distribution in the episodes #3 (p=0.06; p<0.10), #4 (p=0.010; p<0.10), and #5 (p=0.005; p<0.10). However, the values of LR are positive, thus favoring the power-law distribution. Also, all log-normal distribution LR values are negative, indicating that such distribution would be a better distribution for the network. However, the p-values are not statistically significant. Therefore, the paper’s results evidence that the evolution of OilTubs in the Brazilian nicely fits the power-law distribution with few actors accumulating a disproportional number of relationships.
4.3. The Effects of Preferential Attachment on the Network Evolution

Table 4 presents the α results. They indicate the effect of preferential attachment on the network evolution of the internationalizing firm.

Table 4
Preferential attachment results

<table>
<thead>
<tr>
<th>Episodes</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.79</td>
</tr>
<tr>
<td>2</td>
<td>0.88</td>
</tr>
<tr>
<td>3</td>
<td>0.55</td>
</tr>
<tr>
<td>4</td>
<td>0.58</td>
</tr>
<tr>
<td>5</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Source: Authors.

It can be said that preferential attachment has a strong effect on the evolution of OilTubs in the Brazilian market. The values of α are closer to 1 than to zero in the five episodes of the internationalization of the firm (Pham et al., 2015, 2016). This is particularly true in episode #1 (A=0.79) and episode #2 (A=0.88). In these two episodes, OilTubs and Petrobras were the actors with the highest degrees. Although the values of α in episode #3 (A=0.55), episode #4 (A=0.58), and episode #5 (A=0.61) are lower, they also suggest a strong preferential attachment.

Figure 2 also shows that preferential attachment assumes a sub-linear form. The grey shadow, in which the degrees of actors are plotted, is below the full line. It means that the accumulation of relationships by very few actors is not directly proportional to their degree. Even though the OilTubs network evolution fits well with the power-law distribution, it is not an SF network.

Figure 2. Preferential attachment graphs. (a) Episode #1; (b) Episode #2; (c) Episode #3; (d) Episode #4; (e) Episode #5
Source: Authors
in its purest form. In SF networks, the accumulation of relationships is linearly proportional to the actors’ degree (Albert & Barabási, 1999). That is the reason why SF networks are found more easily in technological and biological networks. Actors in such networks have no limits to accumulate relationships (Broido & Clauset, 2018). This assumption does not necessarily hold true in social networks (Gay, 2012). As the network scale matters, the preferential attachment may assume either a sub- or a supra-linear form in social networks (Pham et al., 2015). The results of this paper support this contention.

5. DISCUSSION

Following Cuypers et al. (2020), this paper draws on studies of complex networks to further knowledge in IB. Not only does it apply novel statistical methods to analyze how networks emerge and evolve, but it also builds on the idea of generative mechanisms that drive such an evolutionary process (e.g., Barabási, 2016; Barabási & Albert, 1999; Pham et al., 2015, 2016). In doing so, it finds that the network evolution of the internationalizing firm follows a power-law distribution, being affected by a sub-linear form of preferential attachment.

Specifically, this paper departs from traditional cross-sectional studies by considering direct and indirect relationships formed and terminated between the firm and its counterparts over the years (Ghoshal et al., 2013; Hohenthal et al., 2014). The results show that relationship development does not happen at random. On the contrary, the chances to form and terminate relationships are unequal in the network evolution (Barabási, 2016; Barabási & Albert, 1999). That is why the network evolution does not fit a Poisson distribution but a power-law distribution (Clauset et al., 2009). Few actors situated in the tail of the distribution have a higher likelihood of affecting the network evolution (Andriani & McKelvey, 2009).

This result adds to knowledge in that it shows how the network of the internationalizing firm evolves. By utilizing longitudinal, quantitative data, as suggested by Vahlne and Johanson (2020), this paper captures a phenomenon that falls off the radar of comparative statics: the network evolution of the internationalizing firm (Coviello, 2006; Montoro et al., 2018). Thus, the study draws attention to the importance of a dynamical view to further research on the internationalization of the firm (Welch et al., 2014; 2016; Kurt & Kurt, 2020). Opening such an avenue may avoid Delios’ (2017) premonition that IB will become uninformative and stale with its excessive emphasis on cross-sectional studies.

Another novelty of the paper is that it incorporates actors other than the dyad, i.e., the firm's direct and indirect relationships. As a result, it shows that the network evolution of the internationalizing firm is driven by a generative mechanism operating at higher levels (e.g., whole network) (Barabási, 2016; Barabási & Albert, 1999). The theoretical implication is straightforward: the network evolution of the internationalizing firm does not result from the linear sum of dyadic relationships, i.e., relationships between the internationalizing firm and a particular buyer or supplier. Thus, caution is needed to generalize the results obtained from dyads to the whole network.

This paper also finds that preferential attachment affects the network evolution of the internationalizing firm. Preferential attachment indicates that actors with more relationships have more chances to form new relationships (Pham et al., 2015, 2016). They are preferred for transactions (Barabási, 2016), usually control flows of information and knowledge (Ahuja et al., 2012), and are more resourceful and influential (Lazega & Jourda, 2015). These advantages amplify over time (Aguinis et al., 2016). The more they accumulate prestige, status, and resources, the more they form relationships. The more they form relationships, the more they reap such
advantages (Lazega & Jourdan, 2015). With time, these actors constitute hubs driving the network evolution of the internationalizing firm. Thus, this finding supports the claim that the internationalizing firm is embedded in networks of relationships (Johanson & Vahlne 2009; Vahlne & Johanson, 2020). In doing so, it provides a more nuanced, dynamical view of the internationalizing firm’s network. To our knowledge, this is one of the first papers reporting empirical results indicating that preferential attachment is a relevant generative mechanism for explicating the network evolution of the internationalizing firm.

Although the preferential attachment is found to be a superior mechanism in the network evolution of the internationalizing firm, it does not manifest itself homogeneously as the network evolves (Gay, 2012). Here it assumes a more prominent role at the network onset. Later, preferential attachment slightly loses strength. This finding suggests that costs, managerial capabilities, and node age may have prevented actors from continuously accumulating relationships proportional to their degree (Dagnino et al., 2016; Dahland & Farland, 2013). It also suggests that a competing generative may have come into play in the later episodes of the internationalization of OilTubs. Qualitative data indicate that a potential candidate is node fitness (Pham et al., 2016). Latecomers with valuable intrinsic attributes may have managed to form relationships regardless of their degree. Therefore, this paper suggests that future research should explore this landscape. For example, it can investigate the coexistence of generative mechanisms of the network evolution of the internationalizing firm.

In sum, this paper advances a relational, process-based approach to the internationalization of the firm. It is a series of events that unfold over time towards multiple paths (Welch et al., 2016). Even though current paths restrict future choices, the future is open for the firm and its counterparts (Araujo & Rezende, 2003). Beyond the dyad, various actors form and terminate relationships throughout this process (Ghoshal et al., 2013; Hohenthal et al., 2014; Vahlne & Johanson, 2020; 2021), with very few actors accumulating more relationships. Hubs constituted by such actors may emerge to drive the network evolution of the internationalizing firm. Therefore, preferential attachment appears as a powerful generative mechanism of this evolutionary process.

6. CONCLUSION AND LIMITATIONS

This paper analyzes the network evolution of the internationalizing firm. It concludes that it follows a power-law distribution affected by preferential attachment in its sub-linear form. Nevertheless, this paper has two limitations. First, this paper examines a single case from the gas and oil industry. This industry is highly concentrated worldwide. Besides, a state-owned firm is one of the largest players in the Brazilian market. The industry configuration might have affected the results. Thus, replication of this study in other contexts is needed. Second, even though the dataset is suitable to perform the statistical analysis introduced by Clauset et al. (2009), larger datasets may produce crystal-clearer results for the power-law distribution vis-à-vis competing distributions. Broido and Clauset (2018) remind that social network datasets are usually smaller than technological and biological ones, imposing further constraints on finding pure SF networks in Social Sciences.
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**AUTHOR’S CONTRIBUTION**

Author # 1: Coordination of the research team, research protocol design, data analysis, manuscript writing; Author # 2 – Research protocol design, data collection and analysis, manuscript revision; Author # 3 – Data analysis and manuscript revision; Author # 4 – manuscript writing and revision.

**CONFLICTS OF INTEREST**

The authors declare no conflicts of interests.