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Valuation of a Power Plant with the Real Options Approach

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ABSTRACT: Since the late 1990s, the Brazilian electric power industry has been undergoing significant structural changes, the main objective being to increase competition and attract private investment. Due to this, the National Electric Energy Agency has offered a large number of investment opportunities through auctions of the right to build and operate power plants (mostly hydroelectric) and transmission lines, including small hydroelectric plants.In this article we propose a valuation model for a power generation plant under uncertainty and with the flexibility to choose the optimal power purchase agreement using the real options approach, and then apply this model to the case of a small hydroelectric plant. The results indicate that the flexible project has a value significantly greater than that obtained through traditional discounted cash flow methods.

Keywords: finance; real options; capital budgeting; managerial flexibility; decision analysis.

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1. INTRODUCTION

The Brazilian electric power is going through a process of reforms. Among the objectives are the improvement of the efficiency and self-financing capacity of the sector through the introduction of competition in some market segments, suitable tariffs and regulation by incentives in the sectors that remain natural monopolies. Dueto this, the *Agência Nacional de Energia Elétrica* (ANEEL) has begun to offer a range of investment opportunities, mainly through auctions to build and operate power plants (mostly hydroelectric) and transmission lines, and incentives for investments in small hydroelectric plants (*Pequenas Centrais Hidrelétricas*, or PCHs).

These new investment opportunities in the power sector require investors to analyze the financial feasibility of going ahead with these projects. The problem of pricing power generation projects can be understood as the problem of valuing a concession or risk contract, where the investor holds the operation rights for a finite period and needs to know the value that can be obtained from these rights. In this context, the price of energy both under long-term contracts and in the spot market is influenced by the economy's performance and the supply of the product in the market. There is also flexibility in the mechanism for selling the power generated, either through long-term agreements at fixed price or in the spot market. The choices in this respect can also affect the value of the project. The optimal operational strategy will thus be the one that which maximizes the project's value over its useful lifetime by choosing the best way of selling the power generated at each moment.

In this article we present a model for valuing a power plant concession considering an uncertain energy price, irreversibility of the investments and the existence of management flexibility, through the real options approach. We then apply this model to the case of a small hydropower plant (PCH). The flexibility of managing a power plant project is represented by the option to purchase future cash flows through an investment and by the strategic operational decisions available over the life of the concession. Finally, we illustrate this model with a practical application.

The literature on application of the real options method to value energy assets focuses mainly on the case of thermal plants, due to the management flexibility inherent in these assets. Griffes, Hsu & Kahn (1999) studied the types of real options that can exist in this kind of project, identifying the option to expand, abandon and defer the investment; the option to upgrade and to switch fuels; and the operational flexibility option. Deng, Johnson & Sogomonian (1998) considered that the fuel cost represents the operating overhead variable and the plant is only operated at full load when the energy spot price is higher than the variable cost. Frayer and Uludere (2001) analyzed the value of two power generation assets in the American Northwestern regional market and showed through a real options analysis that a gas-fired plant that only operates during peak demand periods can have a higher value than a coal-fired plant, even though its marginal operating cost is higher.

In Brazil, Castro (2000) studied the value of operational flexibility of a gas-fired plant, incorporating the characteristics of the Brazilian system in the model of Deng, Johnson & Sogomonian and also considering the possibility of bilateral power contracting. Melo (1999) examined the variables that affect energy prices in Brazil, where generation is predominantly hydro powered, concluding that periods of prolonged drought can lead to shortages that tend to raise the prices of energy, while periods of above-average rainfall tend to fill the reservoirs and thus reduce the price. Gomes (2002) studied the dynamic of private investments in thermoelectric generation in Brazil using real options theory to determine the best moment to build a plant, considering the exogenous uncertainty of the expansion of thermal power supply and also when the expansion of supply occurs in response to uncertainties and interactions among agents. However, we did not find any reference in the literature to the application of the real options method to a PCH, probably because the legislation that regulates the construction and operation of these plants is recent, as is a free market for power trading in Brazil as well.

This work is structured as follows. In the first section, aide this introduction, we present the objectives and a review of the literature. In the second section we provide a more detailed view of the electric power market in Brazil and small hydroelectric plants (PCHs), and the different ways of contracting energy that are available. In the third section we propose a model for discrete-time valuation of power generation assets in Brazil through the real options method and then apply this to the case of a PCH and present the results obtained. In the fifth section we conclude.

2. THE ENERGY MARKET IN BRAZIL

Driven by government investments, the Brazilian energy market underwent rapid expansion beginning in the 1960s. This growth was based mainly on the availability of international loans at low interest rates, sectorial tax incentives for financing, a realistic tariff policy and ample availability of water resources at low cost near the consumption centers. The end of external credit and spiraling inflation starting in the late1970s triggered a severe fiscal crisis in the public sector that lasted throughout the following decade, with a steep decline in power generation investments.

In the second half of the 1990s, after the *Plano Real* of 1994 finally stabilized the currency, a process of reforms was undertaken in the country's power sector, with the privatization of assets, implementation of regulatory policies and establishment of the National Electric Energy Agency (ANEEL). The main goal of this process was to concentrate the responsibilities of the government mainly to formulation of energy policies and regulation of the sector, including generation, transmission and distribution. In this new model, new agents were included, such as the free consumer, self-producer and independent power producer. Other measures were implemented to undo vertical integration and introduce free competition in generation and commercialization in order to attract private capital, reduce costs and increase overall efficiency of the system.

One of the areas where incentives for private investments were created was small hydroelectric plants (PCHs). A PCH is defined as a hydro power plant with capacity between 1 MW and 30 MW for independent production or self-production, with a reservoir limited to 3.0 km^2. These projects are typically designed to meet load demand close to industrial production plants, in areas peripheral to the main transmission system and in areas undergoing agricultural expansion, thus promoting development of the more remote regions of the country. Recently, changes have been made in the regulation and incentives for investors in these small projects, which have low environmental impact, in order to add some 5,000 MW to the nation's energy supply in the next ten years. According to ANEEL, bythe end of 2007 there were already 298 PCHs operating in Brazil generating 1,979 MW of power.

Brazil's potential hydropower generation is approximately 260 GW, of which only 28% is being utilized, and of this total, only 1.95% comes from PCHs. Therefore, PCHs are seen as a solution that can be implemented in the short term to increase the country's installed generation capacity. There are currently 76 projects being built, representing 1,278 MW, which are scheduled to come on-line in the short run. This is due to their characteristics, such as shorter construction time, ease of location near consumption centers and less burdensome legal/regulatory requirements.

2.1 Mechanisms for Sale of the Power Generated by PCHs

The energy generated by a PCH can be sold through bilateral power purchase agreements (PPAs), through long-term contracts to the government-controlled Eletrobrás through the Alterative Electric Power Source Incentive Program (*Programa de Incentivo às Fontes Alternativas de Energia Elétrica* – PROINFA) or in the spot market through the Energy Commercialization Chamber (*Câmara de Comercialização de Energia* – CCEE). Under these mechanisms, the sale of power from a PCH can be classified as flexible or inflexible.

The inflexible PCHs are those which sell their energy under long-term contracts, either through PPAs or through PROINFA. They are considered inflexible because their sale prices are established in long-term contracts, and thus are not affected by variations in the spot market price. Flexible PCHs, on the other hand, sell their power directly in the spot market, through the CCEE, and are subject to the price fluctuations of this market.

2.1.1 Power Purchase Agreement - PPA

The bilateral power purchase agreements are negotiated freely between two agents without the interference of the CCEE. They are divided into two sub-categories according to the duration of the agreement: long-term (six months or longer, with the requirement to register the agreement with CCEE) and short-term (less than six months).

The registration of the agreements with the CCEE does not contain information on the prices negotiated; only the amounts contracted in MWh between the parties. These amounts will be accounted for on an hourly basis and modulated by the load level without validations, that is, the data do not need to be equal for the same period.

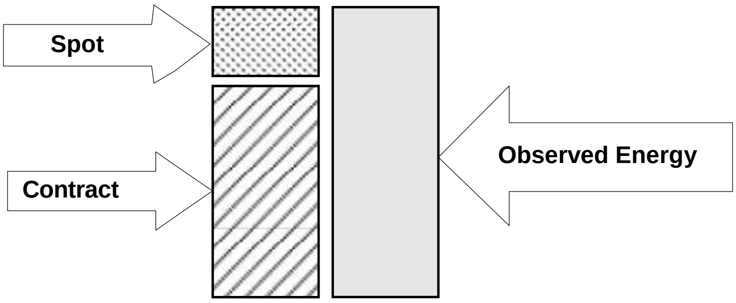
The price of energy in the short-term market is too volatile to efficiently signal the need for new generating capacity. Because of this fact, the model's conception considers that the "engine" of expansion of the system is the willingness to contract part of the demand through bilateral agreements for advance purchase of energy (PPAs). Although PPAs are financial instruments, the regulatory requirement that they be backed by physical generating capacity ensures that the stimulus to such contractual arrangements results in new supply.

2.1.2 PROINFA

Law 10,438, enacted on April 26, 2002, instituted the Alterative Electric Power Source Incentive Program (PROINFA) to increase the participation of energy generated from alternative energy sources, such as biomass, wind and PCHs in the national power grid (*Sistema Interligado Nacional* – SIN). The aim was to contract out 3,300 MW of installed power in a first phase, after which the Ministry of Mines and Energy (MME) would define the amount of renewable energy to be contracted, considering that the impact of contracting alternative energy sources in the formation of the average tariff may not exceed a pre-defined limit in any year, when compared with the growth based exclusively on conventional sources. The economic value of each source, defined by the MME and valid for the first phase of the Program, will be for sale to Eletrobrás, and will have a floor price in the case of PCHs of 70% of the average national tariff to final consumers.

2.1.3 The Energy Commercialization Chamber and Spot Market

The Energy Commercialization Chamber (CCEE) is the venue for processing the accounting of the energy produced and consumed in Brazil, which has a market of about 500 million MWh a year. The accounting in the CCEE considers all the power contracted by agents and all the power effectively verified (consumed or generated), as shown in Figure 1:



Source: Authors
Figure 1 : Accounting of power available in the system

Energy generators, distributors and traders register with the CCEE the amounts of power contracted and the respective measurement data, to enable determination of the differences between what was generated or consumed and what was contracted. This difference is settled monthly in the CCEE at the market price for each regional sub-market (North, South, Southeast and Northeast) and for each level (light, medium and heavy). This is the short-term or spot market.

The price in the spot market is still not determined directly by the law of supply and demand, but it is calculated monthly by means of mathematical models that define the marginal operating cost (MOC), that is, the cost of generating an additional unit of energy over the last unit consumed by the market. Once the MOC is calculated, the Wholesale Energy Market Services Administrator (*Administradora de Serviços do Mercado Atacadista de Energia* – ASMAE) publishes the spot market price, which is equal to the MOC of each of the above four regions (North, South, Southeast and Northeast). This is the price used to settle transactions between market agents complementary to the amounts under bilateral agreements (PPAs). Therefore, the spot price is influenced by the reservoir levels of hydroelectric plants (responsible for approximately 95% of the country's power generation), by the forecast evolution of demand and the current and future availability of plants and transmission lines.

The CCEE is responsible for calculating the spot price in Brazil. For this, it uses dispatch computational tools during the process of optimizing the system. The differentiated spot prices in the four regional sub-markets are defined by transmission restrictions and reflect the marginal cost of the system, considering the generation cost of thermal plants and the system rationing cost. The price is calculated one day in advance and is based on the declarations of availability and operating costs for that date. In countries where the energy sector has been restructured, the determination of the spot price is done through the marginal operating cost (MOC). The use of this method in systems that rely mainly on hydropower, as in Brazil, presents an additional difficulty due to the variability of hydrological conditions.

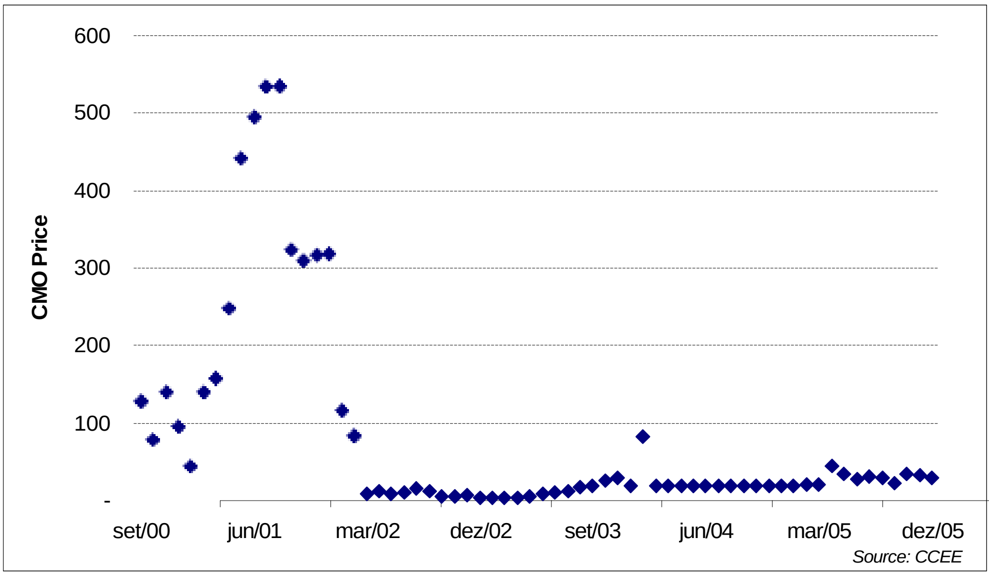


Figure 2 – Evolution of the MOC price from Sept. 2000 to Dec. 2008 ***(Source: CCEE)***

Systems based on hydropower are designed to assure supply even under adverse conditions, which are infrequent. Therefore, most of the time there is excess energy potential, which implies a low MOC. But if a severe drought occurs the MOC can rise sharply, and even reach the rationing cost of the system. Due to the reservoirs' storage capacity, the low cost periods usually occur for various years, separated by periods of high cost caused by droughts or rapid demand growth not backed by spare generating capacity, as shown in Figure 2.

The MOC reflects the dynamic equilibrium between energy supply and demand. It is hard to forecast this price due to uncertainties over the future affluences to the reservoirs, causing a considerable degree of volatility. Besides this, the probability distribution of future prices is very asymmetric.

Table 1 shows the main descriptive statistics of the MOC, broken down by sub-system. It can be seen that the coefficients of asymmetry are much larger than zero, indicating that the distributions of spot market prices have positive symmetry. It can also be seen that the flattening coefficient (kurtosis) is greater than 3 in nearly all cases, indicating a low level of flattening of the distributions and a concentration of values around the mean.

Market	Min	Máx	Mean	Standard Deviation	Asymmetry	Kurtosis	Vol %
Southeast	4.00	684.00	93.91	165.64	2.55	6.08	1.76
South	4.00	430.34	44.60	68.09	3.66	16.75	1.53
Northeast	4.00	684.00	112.80	197.87	1.98	2.58	1.75
North	4.00	684.00	85.37	158.33	2.58	6.21	1.85
Average	4.00	534.74	84.17	136.22	2.21	4.02	1,65

Fonte: CCEE

Table 1 – Descriptive statistics of the MOC (Sept/00 to Dec/05) (Values in R$)

As can be seen in Figure 2, the months when the MOC spiked were precisely those of energy rationing in Brazil (May 2001 to February 2002). Since this period, the regulatory agency (ANEEEL) and the ONS (National System Operator) have been taking preventive measures to avoid new rationing episodes, such as annual revision of future demand projections for the next ten years and tying future sale contracts to the construction of new generation sources, thus giving a greater guarantee of meeting future demand. Table 2 gives the main descriptive statistics of the MOC by subsystem for a more recent period (June 2003 to December 2005), without the impacts of rationing. The coefficients of asymmetry continue being greater than zero, indicating that the distributions of the spot market prices have a tendency for positive symmetry. There is also a noticeable reduction in the kurtosis coefficient, indicating more flattening of the distributions in comparison with the previous series.

Market	Min	Máx	Mean	Standard Deviation	Asymmetry	Kurtosis	Vol %
Sudeste	10,76	50,52	23,08	8,54	1,36	2,07	0,37
Sul	10,76	34,42	22,11	6,06	0,59	(0,27)	0,27
Nordeste	**9,08**	**29,23**	**18,48**	**3,43**	**(0,01)**	**4,73**	**0,19**
Norte	10,55	50,52	22,44	8,30	1,63	3,15	0,37
Média Mercados	10,65	34,72	21,53	5,79	0,58	(0,10)	0,27

Fonte: CCEE

Table 2 – Descriptive statistics of the MOC (June/03 to Dec/05) (Values in R$)

3. THEORETICAL MODEL

We employed the binomial model of Cox, Ross and Rubinstein (1979), assuming that the value of the project *V* follows a Geometric Brownian Motion (GBM) and in each period Δt can assume the value *Vu* , with probability *p,* or *Vd,* with probability *1 – p,* where σ is the project’s volatility, $u = e^{\sigma\sqrt{\Delta t}}$, $d = 1/u$, and $p = \frac{1 + r - d}{u - d}$, and so on, as shown in Figure 3.

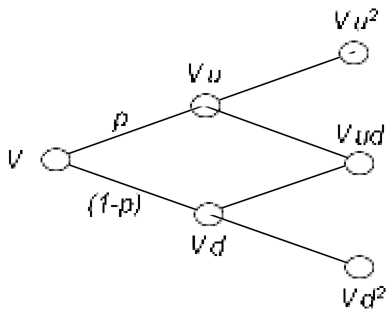


Figure 3 – Binomial Model of Cox et al. (1979)

The project’s value *V* is determined by the discounted cash flow method, projecting the free cash flows over the life of the project. These cash flows are discounted at the risky rate determined by the capital asset pricing model – CAPM (μ). In this static analysis, we consider that the energy will be sold through long-term agreements entered into at the initial moment.

The inclusion of the project’s uncertainties is done by modeling the spot market price, given by the marginal operational cost (MOC), whose initial value will be identical to the energy price for long-term contracts, namely:

- Energy price under long-term agreements

We assume as a base the value established in the first auction of energy generated by new projects (CCEE, 2005), held in December 2005 to define the prices for long-term power purchase agreements. Because this is a long-term contract freely negotiated in the market, we adopt the average price of the auction of R$ 120.00/MWh as the value of the bilateral contract for this analysis, which we assume is constant over the entire project horizon.

- Energy price under short-term agreements (spot market)

The starting value of the long-term agreements tends to reflect the current short-term market price (spot price). Therefore, we also took the average value of the first auction of R$ 120.00/MWh as the base for the initial spot price. To model this variable over the useful life of the project, we considered that the price varies stochastically in time following a GBM. This modeling implies that the price can never be negative and that its volatility is constant in time, as represented by Equation (1).

$$dP = \alpha P dt + \sigma_P P dz \qquad (1)$$

where:

dP is the incremental price variation in the time interval dt,

α is the growth rate of the price in interval dt,

σ_P is the price volatility; and

$d_Z = \varepsilon \sqrt{dt}$, where $\varepsilon \sim N(0,1)$ is the standard Wiener process.

The discretization of this process, utilizing annual periods, provides us the price in each year as a function of the previous year's price.

The project's volatility can be estimated through a Monte Carlo simulation applied to the stochastic cash flow, as proposed by Copeland and Antikarov (2002), and by adopting the modification proposed by Brandão, Dyer and Hahn (2005), considering that the value of the project also follows a GBM. The project modeling can then be done by the model of Cox, Ross and Rubinstein (1979), considering the risk neutral measure as is standard in the options literature, from which the project's management flexibilities will be modeled.

The possibility of operating in both markets (spot and long-term) is a flexibility that can be utilized by investors. Operation of the plant can be started, for example, only with long-term agreements but may not generate the expected revenue. Thus, the project owners can decide to also sell their energy in the spot market to obtain additional revenue. The presence of this flexibility means valuation by the discounted cash flow method may lead the investor to underestimate the real value of the undertaking.

These flexibility options, represented by the option to sell power in the spot or long-term market, however, are exercised on the price, not on the value of the project. Therefore, the basic asset to be modeled is the spot price of energy, not the project's value, as is standard in the literature. This way, the risk neutral measure must consider the risk premium of the energy market, not of the project, and one must deduce this risk premium from the expected growth rate of prices in the modeling of a binomial decision tree.

Since a totally free market for long-term energy prices does not exist, it is impossible to observe the market value or risk premium directly, so it is necessary to do this indirectly. An alterative to solve this problem would be to estimate the value from the process of the cash flows and project value. For example, considering that the single source of the project's uncertainty is the energy price (P), the stochastic evolution of the price through the risk neutral process is represented by:

$$dP = (\alpha - \lambda\sigma_P)Pdt + \sigma_P Pdz \tag{2}$$

where $\lambda\alpha_P$ is the energy risk premium and $\lambda = \rho\left[\frac{E[R_m] - r}{\sigma_m}\right]$ and σ_P are, respectively, the market price of the risk and the volatility of the energy price.

Since the only source of uncertainty of the project is the price of energy, the correlation between the variations in prices and the market will be identical to the correlation between the project and the market. This makes the parameter λ the same both for the energy and for the project. Assuming that the project's present value without options is a reliable estimate of its market value, the risk premium of the project's cash flows can be determined by the CAPM, through $\mu - r = \beta_C(E[R_m] - r)$, and the market price of the risk of revenues λ can be determined through Equation (3):

$$\lambda = \beta_C\left[\frac{E[R_m] - r}{\sigma_C}\right] \tag{3}$$

in which β_C is the Beta of the project and σ_C is its volatility. In this fashion, the energy price risk can be determined by Equation (4):

$$\lambda\sigma_P = \beta_C(E[R_m] - r)\frac{\sigma_P}{\sigma_C} \tag{4}$$

A more detailed analysis of this theme can be found in Hull (2003, pp. 661-667).

Once the project's present value, volatility and risk premium of the revenues[1] are determined, the stochastic distribution of the revenues can be modeled as Brownian motion (GBM) through a binomial model. Once the diffusion model of the project's value is defined and structured, the inclusion of management flexibilities is done by inserting the decision moments where the project's value function will be maximized. At each moment to exercise an option of the project, the optimal decision will be of the type:

Max {value of continuation; value of the option}.

The value of the option will depend on the characteristics of this management flexibility in that period. An abandonment option, for example, can mean that the company will relinquish the future cash flows in favor of a terminal value Ω. An option for expansion can multiply the value of the future cash flows by any factor, deducting the costs of the new investment. A switch option permits exchanging inputs, products or even types of sale contracts. In these cases, the new value of the project from that instant forward, assuming exercise of the option, has to be determined so it can be compared against the project's value

[1] The risk of revenues and power prices are identical, since a PCH's revenue is simply the power price multiplied by the roduction capacity, considered constant.

without exercising the option, and the highest returns have to be chosen as obtained from the decision tree.

4. APPLICATION TO A SMALL HYDROELECTRIC PLANT (PCH)

4.1 Modeling the Basic Asset

In this section we apply the proposed model to a PCH to illustrate its use. We assume that the PCH does not suffer any loss of total generation capacity, which will remain constant over its useful lifetime. We also disregard any need to invest working capital or reinvest in fixed assets, given the small value of such investments.

Table 3 presents the main financial parameters of the project, reflecting the most common characteristics of small hydroelectric projects.

Assumptions	Values
Installed Power	30MW
Cost of the Investment	R$ 138 million
Total Energy Generation per Year	263 GWh
Cost of Generation	R$ 10/MWh
Power Purchase Agreement Price	R$ 120/MWh
Discount Rate (WACC)	10%
Risk Free Rate	4.5%
Useful Life of the PCH	20 years

Table 3 – Parameters adopted to value the PCH

As proposed by the Regional Bank for Development of the Extreme South (*Banco Regional de Desenvolvimento do Extremo Sul* – BRDE)[2] (2002), we assume 10% p/y as an appropriate rate of return for energy generation activities in Brazil, and a risk-free rate of return of 4.5% p/y. We also assume a useful life for the plant of 20 years, considered to be the maximum operating period of a PCH, and that at the end of this period the plant will have no residual value. The installed power represents the operating capacity for generation of hydropower. We consider the case of a PCH in the Northeast region with installed capacity of 30 MW, at the upper limit for this type of power station, with an investment of R$ 138 million, made over the first two years of the project.

With installed capacity of 30 MW and considering for simplification purposes that there will be no loss of capacity or variations in energy demand, there is an average of 8,760 hours per year and total generation of 262,800 MWh, or 263 GWh. Production will only begin two years after the start of construction. Because generation is hydroelectric and due to the small size and simple operation of a PCH, the generation cost basically involves equipment maintenance and the payroll for a small staff. According to the study by the BRDE (2002), we adopt a unitary cost of R$ 10.00/MWh. The cash flow model is shown in Table 4.

Specifications	Observations
Gross revenue	Gross annual revenue: Unit price x Total power generation
- Tax on sales	Cofins/PIS – total rate of 9.25%
= Net revenue	Gross revenue – Taxation
- Cost of generation	Admin. and Oper. O/H – 10 million on/ MWh x Total power generation

[2] BRDE is a regional development bank set up by Brazil's southernmost states of Rio Grande do Sul, Paraná and Santa Catarina.

-	Sector charges	TF SEE/TUSD/ Financial Compensation: 6% of gross revenue
-	Deprecation	Deprecation of the investment over 20 year: R$ 6.9 million per year
=	EBIT	Earnings before interest and tax
-	Income Tax	Income tax and social contribution: 34% on profit
+	Depreciation	Depreciation of the investment over 20 year: 6.9 million per yer
=	Free Cash Flow	Financial flow available to investors

Table 4 – Projected cash flow

In this form, we can express the cash flow model as:

$$FC_t = R_t\left(1-T_i\right)-C_i-(R_t*E)-\left\{\left(R_t\left(1-T_i\right)-C_i-(R_t*E)-D\right)*T_d\right\} \quad (5)$$

where:

R_t: Gross revenue in period t

T_i: Indirect taxes on sale of energy

C_i: Administrative and operational costs in period t

E: Sectorial charges

D: Linear depreciation of the investment over 20 years

T_d: Direct taxes on profits (income tax and social contribution on profit)

For formulation of the basic scenario, we initially only considered the project under conditions of certainty, that is, with the PCH selling its power through long-term PPAs, thus without any type of managerial flexibility. We adopted a risk factor (WACC) of 10%, and computed the present value of the project through the traditional discounted cash flow (DCF) method, using a spreadsheet, according to the data and assumptions presented in this section. The value found was R$ 155.4 million for year 2 (2007), equivalent in year 0 (2005) to R$ 128.4 million. Given that the present value of the net investments is R$ 119.8 million, the net present value (NPV) of the project is positive R$ 8.7 million (Appendix 1).

We determined the project's volatility by a Monte Carlo simulation, where we modeled the price uncertainty of the spot market over the useful life of the project. We ran three simulations with 50,000 iterations each. The results are shown in Table 5.

Simulation Nº	Return (Z)	Volatility (σ)
1	0,1000	0,6998
2	0,1000	0,6997
3	0,1000	0,6998

Table 5 - Results of the Monte Carlo Simulation

The results of the simulation indicate that the price volatility is about σ = 0.70, a bit above the annual volatility of the spot market of $0.19 * \sqrt{12} = 0.66$. To be conservative, we considered the volatility of the Northeast sub-market to be the spot market volatility, as shown in Table 2. Based on a risk-free rate of return of 4.5% p/y, the risk premium of the cash flows is given by $\mu - r = \beta_c\left(E\left[R_m\right]-r\right) = 5.5\%$, and from equation (4) we obtain $\lambda\sigma_P = 0,055 \dfrac{0,66}{0,70} = 5.2\%$.

The parameters of the binomial model are those shown in Tables 3 and 4, besides the volatility and the risk premium of the spot market price, which is deduced from the energy growth rate for the risk-neutral modeling. Because we assume that this growth will be zero over the useful life of the project, the risk-neutral growth will be negative. Figure 4 presents the four first periods. It can be seen that the project's value in year 2 is V_2 = R$ 155.4 million, which corresponds to an initial value V_0 = R$ 128.4 million, identical to the deterministic project's cash flow (cash flows as of year 2 deducting the WACC of 10%).

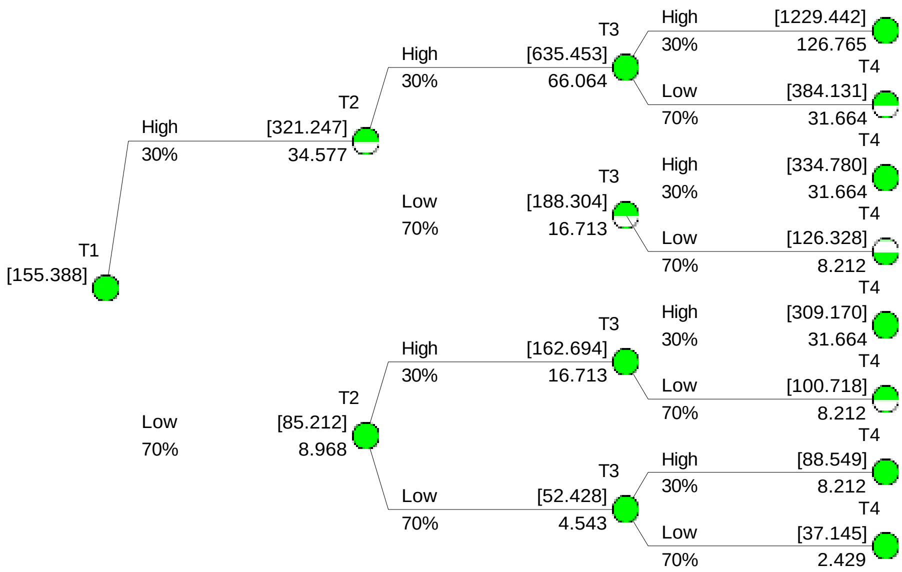


Figure 4 – Decision Tree of the Project

The binomial tree models the stochastic diffusion process of energy prices, by which for each price $P_{t,j}$ in each period t and state j, there corresponds a cash flow $CF_{t,j}$ determined by Equation (5). Then these cash flows are weighted in each period by the risk-neutral probabilities and the present value is deduced using the risk-free rate. The sum of these values then gives the project's value of R$ 155.4 million.

Modeling the Flexibility Options

An investor receiving a concession for a small hydroelectric plant must start the investment immediately after receiving certification by ANEEL, given that the projections of the ONS for verification of the risk of energy rationing (demand > supply) contemplate the project's generation. Thus, there is no flexibility to delay the investment. On the other hand, there is an option to improve the form of contracting the sale of the power, either through the spot market or long-term bilateral contracts.

As observed previously, a PCH has flexibility to sell the energy generated, where it can act through long-term agreements (inflexible) or short-term spot-market arrangements (flexible). As seen, the spot market price is extremely volatile and the long-term market, either through PPAs or PROINFA, has practically no volatility, since the prices are pre-established for the future periods, and in general are only adjusted for inflation.

Therefore, a PCH that is operating in the spot market at prices below that of a five-year PPA will be able to contract all its output in the long-term market and generate higher revenue. Similarly, a PCH that is selling under a five-year PPA that expires today at a price below the spot price can shift over to selling all its output in the spot market and also obtain higher revenue.

We considered that the flexibility to operate in the short-term or long-term market is a contractual option that will be exercised in optimal form. The choice is between selling in the spot market or through a five-year PPA. We chose the PPA instrument and five-year period using as an assumption the first auction of energy generated by new projects (CCEE, 2005), because this represents the main long-term energy trading mechanism. In the medium term we chose as the assumption the bilateral contracts actually reached between operators.

In the case under analysis, we considered that the market decisions (options) will be taken in four periods during the project: year 0 (initial), year 5, year 10 and year 15. These decisions are spread out at five-year intervals considering the length of the long-term contracts and that these will not be breached.

Table 6 summarizes the contracting option:

Contracting Option
Option 0: Year 0
Option 1: Year 5
Option 2: Year 10
Option 3: Year 15
Benefit: Cash flow for the energy price of the chosen market (LT or Spot)

Table 6 – Contracting Option of the PCH

The optimal choice at each option point is made by comparing the value of continuing in the spot market against that of entering into a long-term (five-year) contract:

$$\mathrm{Max}\ \{VS_t, VLT_t\}, \quad t = 0,\ 5,\ 10,\ 15$$

where: VS_t: Value of continuing to contract in the spot market in period t

VLT_t : Value of long-term contracting in period t

Hence, there are four contracting options, in years 0, 5, 10 and 15, representing the opportunities to switch between the spot market and long-term contracts.

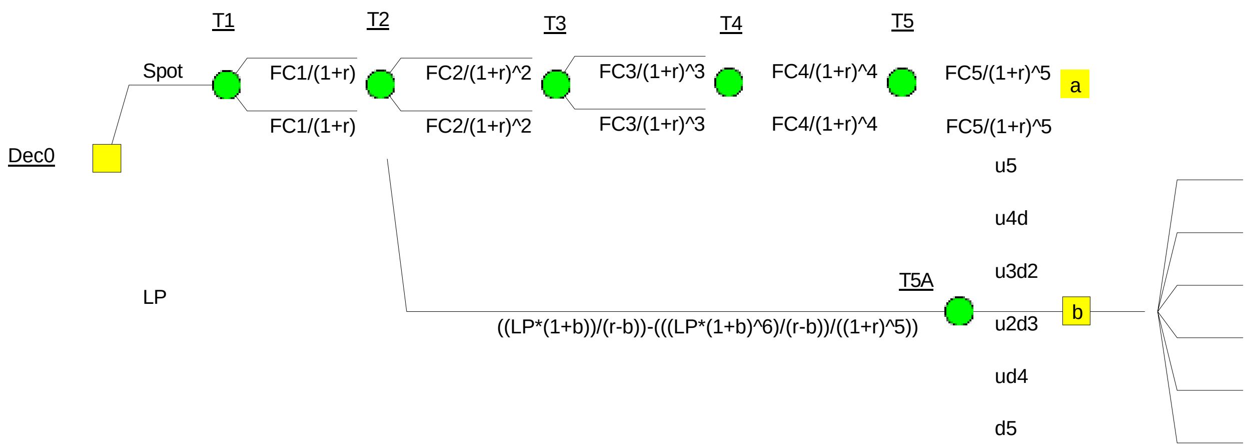


Figure 5 – Decision tree with contracting option in year 0

The first option between the spot market or long-term contracting occurs at the start of the project (year 0). After this decision node the tree is partitioned into two different paths, represented by the two branches. The first branch represents the option for the spot market, where the energy price will vary stochastically each year according to Brownian motion and the parameters defined previously. This diffusion process is represented by the binomial model of five periods in the upper branch of the tree, as illustrated in Figure 5, where the value of the project's first five years is given by $\sum_{i=1}^{5} \frac{CF_i}{(1+r)^i}$, where CF_i are the free cash flows in each period, according to Equation (5), and r is the risk-free rate. The second branch represents the option for a five-year contract at a fixed price, and the expression on the lower branch of the decision tree corresponds to the present value of the cash flow of an annuity with five-year term, where b is the opposite of the risk premium of the cash flows. The uncertainty node T5A at the end of the lower branch of Figure 5 generates the probability distribution of the spot price in year 5, identical to that generated by the uncertainty nodes of the upper branch, so that this information is available for making a decision in year 5, regardless of the optimal path chosen in year 0. In both cases an adjustment is made in the expected growth rate by deducing the energy risk premium, to enable risk-neutral valuation.[3]

For the decision in year 5, each path will again be partitioned into two more paths and successively thereafter for the decisions in years 10 and 15. If the option is for the spot market in year 0, in year 5 the decision can be maintained or replaced by long-term contracts for the next five years, as shown in Figure 6.

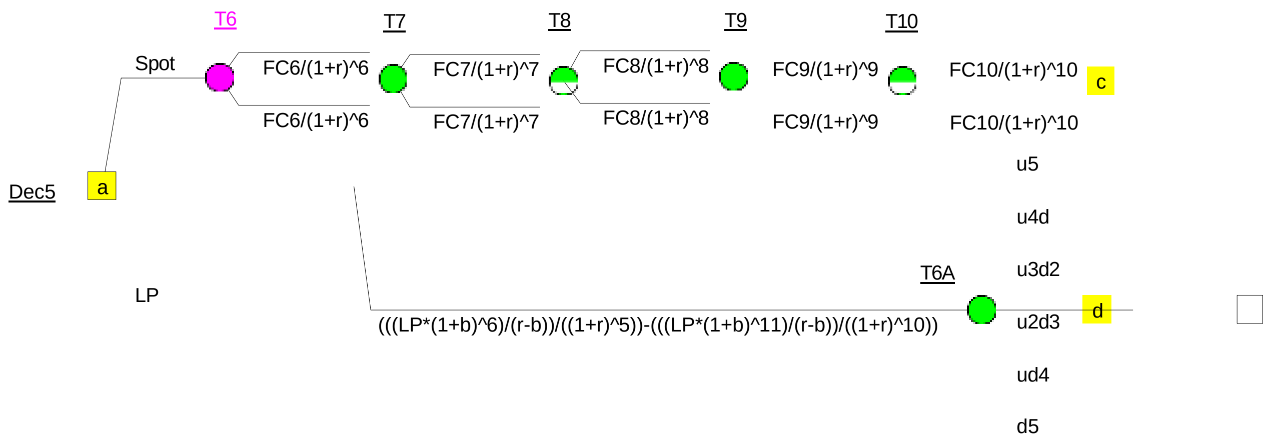


Figure 6 – Decision tree with contracting option in year 5

In years 10 and 15 there are again options to choose the optimal contracting mechanism for the following five years, until year 20, the last of the project's useful life. Each of these decisions represents a switch option, permitting choice of the alternative that offers the best expected value at each of the decision moments.

The results indicate that the project's value, with the flexibility to choose between the spot market and long-term contracting increases from R$ 128.4 million to R$ 173.3 million

[3] For the risk-neutral valuation of the five years of contracting under a long-term agreement, we used the annuity formula of a flow with constant growth rate (equal to zero in this case) less the risk premium of the flows. Since the relation between the power price and the cash flows, according to Equation (5), it is not exactly constant due to the presence of fixed costs, the risk premium of the project's flows (-*b*) is slightly different from the risk premium of the energy.

(R$ 209.7 million in year 2 deducting the WACC of 10%). Figure 7 shows the first four periods of the decision tree.

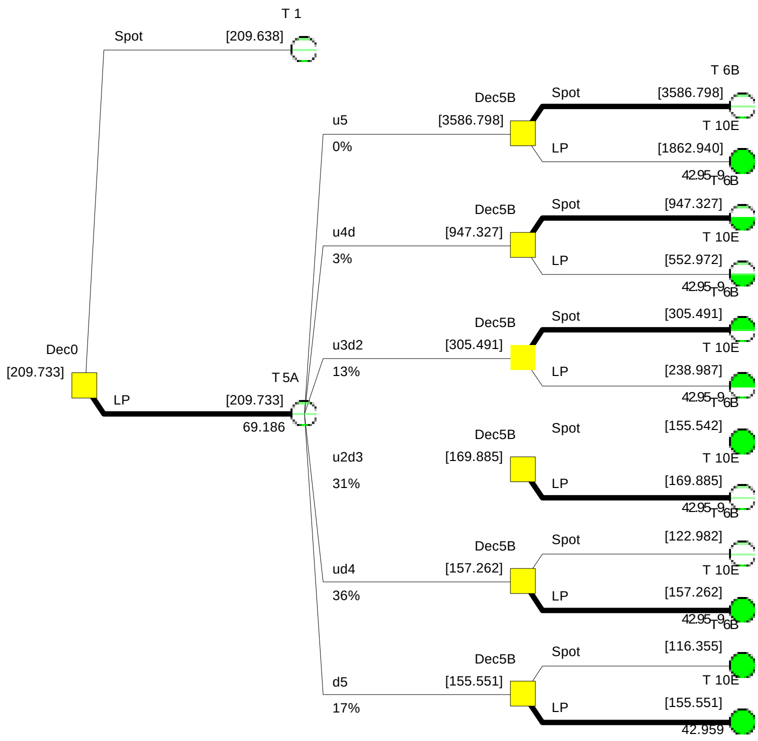


Figure 7 – Value of the project with the contracting option

The sensitivity analysis of the project in relation to the volatility of the spot-market price is shown in Figure 8, where the frontiers are indicated where shifts in the company's optimal strategy occur. The results show, as expected, that the project's value increases with the price volatility.

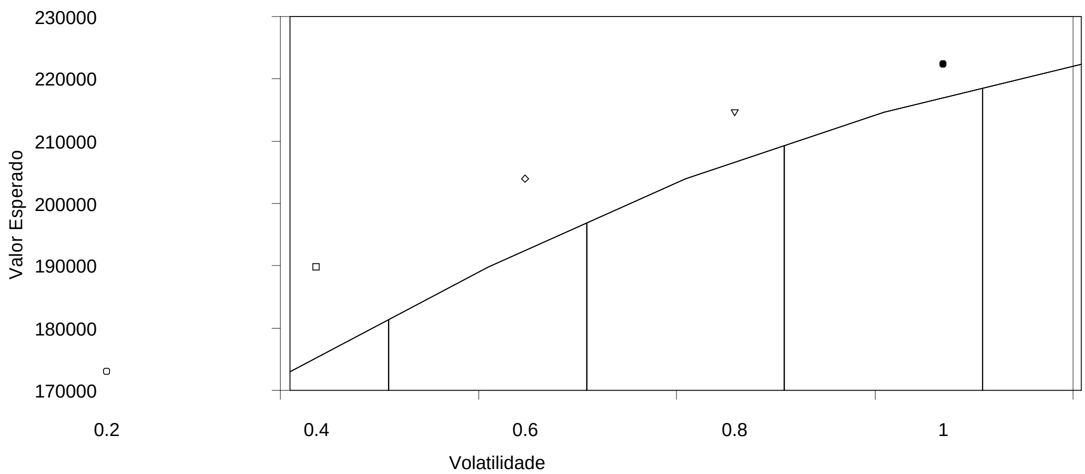


Figure 8 – Value of the Project: Sensitivity to spot-price volatility

We also analyzed the project's sensitivity to the initial spot-market price of energy (Figure 9). The results indicate that the project's value also increases with increasing initial energy price in the spot market.

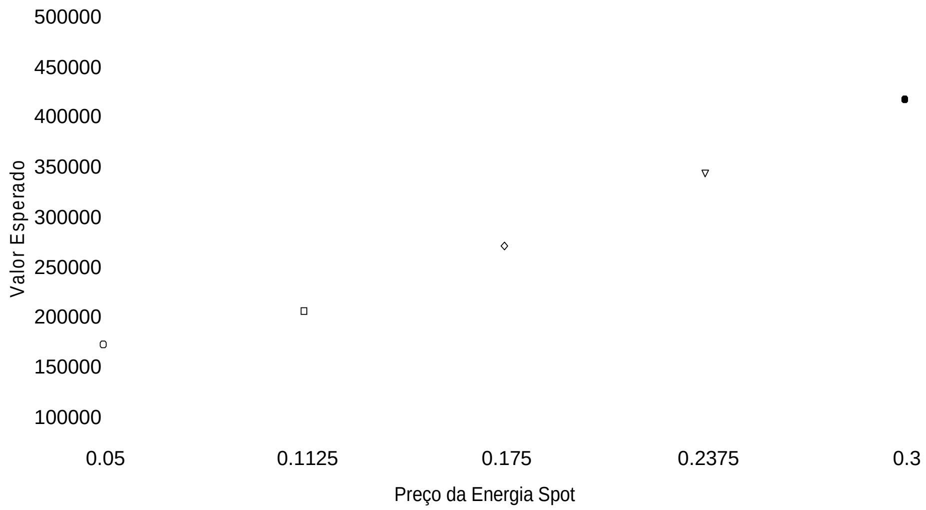


Figure 9 – Value of the Project: Sensitivity to the initial spot price

5. CONCLUSIONS AND RECOMMENDATIONS

In this article we analyzed the case of a power generation company that has the flexibility to choose the market in which it will sell the energy generated by a project, either through a long-term agreement at a fixed price or in the spot market with floating prices. To do this we used the real options method, assuming that the company periodically has the option to change the market decision taken previously to maximize the value of the undertaking.

The methodology proposed in this article is innovative, inasmuch as we did not find any study of a similar situation in the national or international literature. We attribute this to the peculiarities of the Brazilian energy market, the specific regulations on the operation of small hydroelectric plants (PCHs) and the relatively recent development of tools to value flexible options in projects.

This model was applied to a hypothetical PCH in the Brazilian market using actual market data. The results indicate that with the presence of management flexibility regarding the market where the power will be sold, the project's value increases from R$ 128.4 million to R$ 173.3 million in relation to the value without flexibility, a gain of about 35%. This increment is significant and is not captured by the discounted cash flow methods traditionally used for this type of analysis. We conclude that for projects with these characteristics, the best way to determine the potential creation of value for the investor is the real options method.

Despite the significance of the results obtained, the model presented here may not be applicable to all situations as some of the assumptions adopted to model the market variables, such as that the energy price follows a Geometric Brownian Motion diffusion process. For commodities in general, it is common in the literature to assume that prices tend to revert to the mean. In the Brazilian case, however, given that the price of energy is strongly influenced not only by demand, but also by the country's hydrologic conditions due to the predominance of hydropower in the country's energy matrix, the definition of the best stochastic model to employ still needs further study.

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APPENDIX 1: VALUATION OF THE PCH BY DISCOUNTED CASH FLOW

Cash Flow (R$)		2005	2006	2007	2008	2009	...	2027
Installed Capacity (MW)			-	-	30	30	30	30
Power Supply (MWh)			-	-	262.800	262.800	262.800	262.800
Final Sale Price (R$/MWh)			120	120	120	120	120	120
GROSS REVENUE					31.536	31.536	31.536	31.536
(-) TAXES					(2.917)	(2.917)	(2.917)	(2.917)
Cofins	*- Rate (%)*	*7,6%*			(2.397)	(2.397)	(2.397)	(2.397)
Pis/Pasep	*- Rate (%)*	*1,65%*			(520)	(520)	(520)	(520)
NET REVENUE					28.619	28.619	28.619	28.619
(-) Cost of Generation	*Unit Cost*	*10*	*(R$/MWh)*		(2.628)	(2.628)	(2.628)	(2.628)
(-) Misc Fees	*- Rate (%)*	*6%*			(1.892)	(1.892)	(1.892)	(1.892)
EBITDA					24.099	24.099	24.099	24.099
(-) Depreciation					(6.900)	(6.900)	(6.900)	(6.900)
EBIT					17.199	17.199	17.199	17.199
(-) IR/CSSL	*- Rate (%)*	*34%*			(5.848)	(5.848)	(5.848)	(5.848)
EARNINGS AFTER TAXES					11.351	11.351	11.351	11.351
(-) Investments			69.000	69.000				
(+) Depreciation					6.900	6.900	6.900	6.900
FREE CASH FLOW			(69.000)	(69.000)	18.251	18.251	18.251	18.251

Valuation (R$)	WACC	10%
	2005	**2007**
PV of Cash Flows	**128.415**	**155.388**
PV of Investments	**(119.752)**	
Project NPV	**8.663**	