

## Analysis of the non-linear effect of net equity in the pricing of stock investment funds

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### ABSTRACT

This paper uses the *Capital Asset Pricing Model* (CAPM), in its canonic version and with non-linear extensions, aiming at pricing a panel of 75 stock investment funds in Brazil, throughout the last 11 years. The result suggests that the linear version of said framework is not capable of pricing or forecasting actual returns of funds which have high net equity (NE) and outperformance, with respect to the index of the São Paulo Stock Exchange (Ibovespa), corroborating previous evidence. The non-linear version with thresholds based on the NE seems to deal better with the issue of significant Jensen's alphas, although it is statistically indicated only for a few funds with high NE, but low outperformance. This is evidence that, even though size influences the management and, possibly, the performance of a fund, the pricing modeling of such effect should be made linearly.

**Keywords:** Threshold capital asset pricing model (TCAPM); Stock investment funds in Brazil; net equity.

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## 1. INTRODUCTION

Currently, it is estimated that approximately 90% of the volume of financial transactions of investments or financing of companies worldwide are directly associated with the various types of mutual funds. In Brazil, the financial volume in the funds industry exceeds 60% of the total values raised in the national financial system, considering term deposits, demand deposits and savings accounts.

Such empirical evidence concerning the relevance of said market in the current economic scenario has as theoretical grounds the power of diversification, as addressed 60 years ago in the framework of Modern Financial Theory.

With respect to stock investment funds, despite the evolution of this category, there has not been considerable developments in the literature in terms of theoretical modeling of pricing and forecast of returns of dynamic portfolios composed by such funds. Accordingly, a natural question would be one concerning the capacity of the frameworks traditionally used for pricing shares to incorporate the sources of risk and the specificities of such market.

According to Sharpe (1991), the gross returns obtained by the funds before the deductions associated with the transaction costs are such that investors shall not have outperformance in relation to the benchmark, in case they invest in strategies containing only passive funds indexed to such benchmark, i.e., the Jensen's alphas associated with the pricing model in question would be null. In this context, known as arithmetic of active management, the Jensen's alpha shall be null, with respect to the category of investment funds with active management.

Many variables associated with the expertise of the manager, or with the size of the fund, among others, could be relevant in the explanation of the performance observed in a category of investment funds. According to Chen et al. (2004), for a panel of American funds, from 1962 to 1999, even after the due control for a wide set of variables, there is a negative correlation caused by the size of the fund in its gross performance, possibly associated with the effects of liquidity and internal organization.

Specifically for Brazil, Matos and Rocha (2009) identified a pattern of outperformance associated with the size. According to the authors, “[...] the performance of the pricing models depends on a pattern of the stock investment funds in Brazil, ... the factor models capture better than the CAPM the risks associated with investment funds with higher net equity (NE) and with larger performance gap in relation to the Ibovespa. This improvement, however, does not seem to be sufficient, which may be evidence of the need to construct factor models in the manner of Fama and French, which accommodate the specific anomalies of the investment funds market.”

This paper is in line with the discussion instigated in Chen et al. (2004) and it aims at continuing Matos and Rocha (2009) when dealing with evidence on the relevance of allowing distinct dynamic behaviors for the return of an investment fund, depending on the regime, associated with its size, in which said fund is found in time. Thus, the use of a nonlinear framework for the CAPM, the Threshold CAPM (TCAPM), is suggested here, in the manner, for instance, of the TAR (Threshold Autoregressive) model initially proposed by Tong (1978) and Tong and Lim (1980), in the pricing and forecast of a panel with 75 stock investment funds, in

Brazil, for the period from January 1998 to December 2008. The objective is to measure the degree of robustness of the evidence previously obtained, in addition to evidencing the existence of a common pattern for the investment funds causing them to be modeled in a more adequate manner, from threshold effects caused by the levels of net equity (NE) of the respective funds.

The main results suggest that the incorporation of the non-linearity seems to be relevant when dealing better with the disturbance generated by significant Jensen's alphas, considering the returns after administrative costs. However, there are restrictions as to the use of this non-linear framework, being well specified only for some funds with the characteristics of high NE and low performance, suggesting that it may be necessary to develop linear modeling, with specific risk factors for investment funds capable of capturing such “anomalies” or patterns.

This work is structured as follows: in the next section, brief comments are made on the investment funds in Brazil; in section 3, a literature review is made, describing in details the empirical exercise in section 4; in the two last sections, the results are discussed and final considerations are made.

## **2. INVESTMENT FUNDS IN BRAZIL**

In Brazil, although the first investment funds appeared in the 1950s, more precisely with the “Crescinco Fund”, in 1957, only with the Stock Markets Law (Law nº. 4.728) this sector gained some force, with a second significant stimulus taking place in the 1970s, when more regulation was provided by resolution 145 of the Central Bank. Currently, it works under the authorization of the CVM (Securities Commission), agency responsible for its regulation and supervision, seeking to protect the investor, by means of CVM Instruction No. 409, of Aug 18, 2004.

Also known as mutual funds [fundos mútuos] in Brazil and as mutual fund or open-end company in the United States, such legal organizations, in the form of an entity under co-ownership of investors, have a specific investment policy in accordance with their own articles of incorporation, which also include the rights and duties of members, rules on the system for collection of fees and aspects related to its corporate organization, such as management of the assets and liabilities, the administration and custody of assets which compose its portfolio.

The specialized management of investment funds tends to reduce the information asymmetry existing in the market where they act, in addition to offering a good investment option to those who choose a passive strategy of portfolio rebalancing, providing small investors with access to better market conditions.

With respect to the category of stock investment funds, it can be observed that the average NE in a cross-section of 75 funds, during the period from 1998 to 2008, oscillates from BRL 2 million to BRL 1 billion. On the time dimension, some interesting statistics, data from the National Association of Investment Banks (Anbid) indicate that the Industry of Funds in Brazil has shown successive increases in its aggregate NE, reaching its apex, in 2007, with net funds raised of BRL 50.8 billion and with approximate equity of BRL 1.16 trillion. In 2008, the scenario was quite different, with the funds industry having been significantly affected by the international financial crisis, showing a retraction of BRL 57 billion, ending the year with

approximate equity of BRL 1.13 trillion, a 5.01% reduction to NE, according to description in Table 1.

**Table 1: Funds Raised - Brazilian Funds Industry**

	<b>Net funds raised (in BRL billion)</b>	<b>Net Equity (in BRL billion)</b>	<b>Funds raised as % of the NE</b>
2002	-65.32	356.05	-18.97%
2003	61.56	515.95	17.29%
2004	6.76	613.70	1.31%
2005	19.46	739.17	3.17%
2006	68.42	939.35	9.26%
2007	50.42	1.157.91	5.37%
2008*	-56.86	1.135.37	-5.01%

\* Until December 15, 2008

Also in accordance with Anbid, until mid-September 2009, the net funds raised throughout that year were BRL 1.3 trillion, evidencing the recovery of this “industry”.

To obtain greater detail on such market, it is indispensable to read a very informative recent survey, developed by Varga and Wengert (2009), where they describe the history and the evolution of such industry in Brazil, addressing the regulatory aspects, the types of agents involved and the economic environment. Source of innumerable statistics, we emphasize the fact that there are in Brazil over eight thousand investment funds, until the end of 2008, in addition to their significant relevance in this market in Latin America, occupying the first place in the region and the eleventh place when compared with all other countries.

### 3. REVIEW OF LITERATURE ON PRICING OF ASSETS

#### 3.1. RISK-RETURN TRADE-OFF

There seems to be a consensus in the financial sector on the fact that the biggest challenge in the Theory of Asset Pricing is associated with the capacity to propose micro-based frameworks, accommodating the main empirical evidence in the most diverse financial markets, in the time dimension or cross-section. This mainstream can be considered recent, being the result of discussions arising from the first studies developed in Markowitz (1952), under which the bases of the Modern Theory of Portfolios were established, in the 1950s. There were innumerable developments and contributions of this framework, from the theoretical support concerning the capacity of risk diversification to the specification of the preferences of an investor, whose arguments would be associated only with the two first central moments of the distribution of the financial assets' returns.

Approximately one decade later, Sharpe (1964), Lintner (1965) and Mossin (1966) derived in classic works the framework which would give rise to the Capital Asset Pricing Model, or CAPM. Even though it is considered the most traditional, most known and also most used asset pricing model in the financial market worldwide, a vast range of criticism and limitations can be pointed out (see Roll (1977), Stambaugh (1982), Roll and Ross (1995), Kandell and Stambaugh (1995), among others), in addition to robust empirical evidence against the empirical success of this framework, such as the size effect in Banz (1981) or the contradictory positive correlation between leverage and average return reported in Bhandari (1988).

A third moment of advancement can be characterized by the development of approaches based on optimum decisions of consumption and savings, the well-known Capital Consumption Asset Pricing Model (CCAPM), from works such as Lucas (1978), Breeden (1979) and Mehra and Prescott (1985).

According to Cochrane (1990), it would be possible to summarize this complex range of models comprised in the theory of pricing in a simple pair of correlations, where one of them describes the empirical representation related to the cost of the asset itself, while the other one specifies which economic assumptions are being made. In this classic survey, the author also formalizes the connection between the pricing approach via Stochastic Discount Factor, a random variable which correctly discounts the flow of payoffs of an asset, mean variance frontier, beta representations and linear factor models.

Based on this connected set of frameworks and representations, there have been countless attempts, with or without success, to accommodate puzzles and to forecast prices of financial assets.

Considering the category of fixed income assets, the factors approach used in Fama and French (1993) should be mentioned, which aimed at capturing the differences among return characteristics of long and short term bonds. We should emphasize the literature which aims at establishing the correct and justified price for the category of derivatives, interesting and useful assets for hedge and speculative transactions, either using trivial frameworks, such as the binomial, or even restricted access, by means of the use of stochastic calculation in Black and Scholes (1973).

Specifically concerning stocks, one of the vastest literatures in finances on the most diverse approaches or objectives is started. It is possible to summarize the main schools of thought which serve as basis for the theoretical and empirical works, which are associated with the attempt to identify variables that: *i*) have an influence in the pricing and forecasting capacity of the models, such as size, leverage, ratio between net equity and market value (book-to-market) or dividend and price, thus generating patterns which corroborate the perception and the theoretical results of models or “seal” the end of their application; *ii*) have explanatory power in commonly linear approaches in the cross-section or time dimension.

The following articles are among those which are more strongly mentioned and followed in this area: Ross (1976) with the Arbitrage Pricing Theory (APT), Fama and French (1992), (1993), (1995) e (1996) and Carhart (1997), exploring factor models and size effects, book-to-market, market risk and momentum.

Cochrane (2001, 2006) is an excellent source on pricing models for individual stocks, being possible to evidence that, although extremely legitimate, such extreme concern can be treated as a secondary issue, other markets, mainly the investment funds market, admittedly relevant and with one of the highest potentials for growth, as defended theoretically in Markowitz (1952), by means of the supremacy of passive strategies, and argued in Vargas and Leal (2006), according to which one of the most important aspects of the financial theories is the efficient management of portfolios.

In light of this evidence on a type of absence of specific literature addressing mutual funds for investment in stocks, or even investment funds in other categories, it is essential that we ask

what is being developed in terms of theoretical modeling aiming at pricing or forecasting the returns of such dynamic portfolios. Would the linear frameworks of factors traditionally used in the pricing of individual stocks be capable of incorporating the sources of risk and the specificities of this market?

Among the few articles found in the international literature, the recent work of Fama and French (2010) stands out, which, when comparing portfolios formed by traditional funds and market portfolios, evidences, by means of simulation via bootstrap, that only a few funds would be capable of generating returns compatible with market benchmarks to the point of offsetting the costs of active management which is a characteristic of the funds. Such scarcity of articles addressing modeling of funds is even stronger with respect to analysis of national markets other than the American market.

### 3.2. CAPITAL ASSET PRICING MODEL: APPLICATIONS AND EXTENSIONS

The main result of the CAPM is that the excess return for a certain financial asset would be associated linearly with the excess market return, according to the following equation:

$$E[R^i] - R^f = \beta^i \cdot [E(R^m) - R^f] \quad (1)$$

where  $R^i$  consists of the actual return of the asset  $i$ ;  $R^f$  is the actual return of the asset seen as risk-free;  $\beta^i$  is the sensitivity coefficient of the asset  $i$  in relation to the market portfolio;  $R^m$  consists of the actual return of the market portfolio and  $E[.]$  is the expected value operator.

Among the vast literature which applies this framework to the Brazilian market, one of the most interesting studies would be Barros, Picanço and Costa Jr. (1998), according to which there would be in Brazil the so-called *Golden Opportunity*, which, according to Haugen (1997), provides in the long term a good yield with a low level of risk, contradicting what is established in the traditional version of the CAPM. Araújo et al. (2006) seek in their work, by means of the methodology proposed by Hou (2002), a proxy which explains the return of asset portfolios in the context of the CAPM, obtaining as a result that the Ibovespa, although not satisfying the conditions of validity of the CAPM, is more efficient as a reasonable measure for the market portfolio than the alternative proposed by Hou (2002).<sup>i</sup> Chague and Bueno (2008) contribute by testing the CAPM, as well as the framework of three factors for a panel of stocks, during the period from 1999 to 2007. Among the various results reported, the authors analyze the significance associated with the nullity of the intercepts, as a form of efficiency of the models proposed, which is in line, on this aspect, with Matos and Rocha (2009).

From a more critical and theoretical standpoint, despite its stronger premises, with emphasis on the homogeneous expectations among investors, since its formulation, various researchers have sought to test the CAPM empirically, defending it or asserting that it is not valid. In addition to the already traditional criticism of Roll, according to which the model would not be testable due to the fact that the market portfolio is not observable<sup>ii</sup>, and the criticism of Hansen and Richard, which questions the sets of information of agents and their respective

expectations, there is countless evidence indicating the flawed specification of the CAPM, introducing other important factors in the determination of the expected return of the assets.

Thus, even accepting the market indices as proxies for such market portfolio, the research analyzes the behavior of systematic anomalies detected in the formation of prices of the assets and not explained by the CAPM, with an extensive literature being developed in the Theory of Asset Pricing, aiming at relaxing the hypotheses of the original framework. It is possible to evidence an interesting degree of robustness of the CAPM to the flexibilization of some of its hypotheses and identify other explaining factors.

In this context, in 1976, Ross developed an alternative modeling based on the hypothesis of non-arbitrage, the Arbitrage Pricing Model (APT), which supports the formation of the equilibrium prices of the assets as a result of the systematic influences of factors of economic nature, even if they are not directly observable. Subsequently, various factor models were developed, with prominence for Fama and French (1992 and 1993), which evidenced that the effects book value/market value and the size of the firm absorb the majority of the anomalies which invalidate the CAPM. Such authors suggest an alternative model, described below, which includes, in addition to the market factor, other two factors: the SMB, related to the size of the firm and the HML, related to the book value/market value.

In Fama and French (1998), they confirm that such effects are present in various countries in the period from 1975 to 1995. Jegadeesh and Titman (1993) identified the Momentum effect, according to which stocks with low returns in the last 3 to 12 months tend to have worse performances in the next three to twelve months, while stocks with extraordinary returns in the same period would tend to keep high returns for the next months. Innumerable frameworks, in their great majority *ad hoc*, have been developed with the intention of modeling beyond risk premiums or size effect, or relevant aspects of the stock market, such as liquidity premium.

### 3.3. THIS WORK AND THE RELATED LITERATURE

As evidenced in the literature on stocks, it seems that it is quite pertinent to raise questions on anomalies or patterns specific to the funds market associated with the “size” of such funds. Chen et al. (2004) describe one of the main pieces of evidence in the American funds market on the influence of the size in the management or even in the performance of the funds. Accordingly, would the existing theoretical models or frameworks be capable of accommodating such patterns?

In one of the few empirical works developed for the stock funds market in Brazil, the evidence obtained in Varga and Wengert (2003) from a wide cross-section of over one thousand funds from 1997 to 2003 stands out, according to which benchmarks of the Brazilian market, such as Ibovespa, inflation, CDI and foreign exchange transactions BRL/USD, would be capable of explaining approximately 80% of the returns of more than half of such funds characterized by active management.

With respect to evidence of patterns in the cross-section dimension, Matos and Rocha (2009) analyzed the capacity of pricing and forecasting the actual returns of the main stock investment funds in the Brazilian market, during the period from 1997 to 2006, using the (CAPM) and factor models in the manner of Fama and French (1993). The main contribution

consists of the result that the performance of the CAPM could be associated with the net equity and outperformance in relation to the Ibovespa of such funds, and such performance would be “lower” as both measures of the funds had higher values.

The results of said authors indicate that there is evidence that, although the factor models are more efficient in capturing better than the CAPM the risks associated with investment funds, since they were not developed specifically for the funds market, this improvement, however, does not seem to be sufficient, there being a need to construct frameworks, linear or otherwise, which accommodate the specific anomalies of the investment funds market, especially with respect to the size and outperformance effects.

Even though there is an unquestionable relevance in deriving models with such purpose, what has been developed, for example, in Matos and Silva (2010), is interesting when analyzing the performances of such models on another aspect, different to those mentioned, all characterized by the direct and linear incorporation of new risk factors.

The discussion could thus have two aspects: which risk factors should be taken into consideration and in what manner should they be incorporated into the theoretical framework. To this effect, the main aspect in this work shall be to analyze the impact of the net equity of a stock investment fund in a traditional model's capacity to price it. This choice is not only a consequence of the evidence previously mentioned, but also a result of the evident relevance of this accounting variable in the management of such funds, mainly those which invest in assets in the stock market.

A recent example of such impact consists of bad performances of multi-market funds, aggravated throughout the year 2008, by virtue of the simultaneous and significant exit of investors, i.e., of the strong reduction of NE. Also in this context, as a violation of the Free Formation of Portfolio, certain assets of the market would only be accessible for investors with quite high interests, such as BRL 500,000.00 interests, limiting or at least restricting acquisition by small investment funds.

The incorporation of this accounting effect in the modeling follows from the evidence that it can be quite useful to establish different regimes and allow distinct dynamic behaviors for the time series of economic variables with non-linear behavior, since statistical properties may vary between regimes.

The model with threshold effect consists of a highly popularized modeling tool in this situation, and the TAR model initially proposed by Tong (1978) and Tong and Lim (1980) was one of the pioneering models.<sup>iii</sup> The central idea of the TAR model is to modify the parameters of the autoregressive model in accordance with the value of an observed variable, known as the decision variable or threshold variable. Among the applications of threshold effect in the literature of empirical finances, the works of Gouriéroux and Monfort (1992), Akdeniz et al. (2003) stand out.

Gouriéroux and Monfort (1992), using ARCH and GARCH models, allow a possible negative correlation between risk and return when the excess return in the market is negative. With such threshold effect, they observed that the CAPM model more is adequate for markets with downward trend when compared to those with upward trend. Akdeniz et al. (2003) also proved a better adjustment of the CAPM model, in terms of the dimension of the pricing errors,



when the market betas change in accordance with the economic scenario - by means of threshold effects.

In this context, methodologically in line with Akdeniz et al. (2003), but for a panel with 75 stock funds in Brazil, from 1998 to 2008, this paper aims at evidencing whether or not there is a robustness of the evidence obtained in Matos and Rocha (2009), where, for a sample of 18 stock investment funds, there would be a need to derive models more advanced than the CAPM to price funds with high NE and outperformance. We also analyze the evidence of some pattern or characteristic common to the investment funds which could cause them to be more adequately priced by a non-linear threshold approach, having the corresponding fund's NE as decision variable, whose threshold is determined endogenously. The goal is to contribute with such literature, by capturing a non-linear effect of the accounting variable, from the hypothesis that “very high values” of NE can interfere with the management or the performance of the fund, and especially with the models' capacity to price it.

#### **4. EMPIRICAL EXERCISE**

##### **4.1. DATABASE**

The empirical exercise will be based on a simple statistical test, according to which it will be possible to evidence whether the extension incorporated into the traditional CAPM has a better behavior in the pricing and in the forecasting of return of stock investment funds of the Brazilian market, and, in case it has, for what class of investment funds. In view of the goals established in this article, the data required to be used were extracted from the database system of the website Fortuna, specialized in investment funds in Brazil, and of the Brazilian Institute of Geography and Statistics (IBGE). From the first source, the time series of nominal return and net equity of all stock investment funds classified by Anbid, and in activity in Brazil from January 1998 to December 2008, were extracted, thus comprising 132 observations throughout time for 75 investment funds, whose names and respective codes are listed in Table 2.

Table 2

Basic Information on Stock Investment Funds in Brazil (Panel with 75 Funds, from 1998.1 to 2008.12) <sup>a, b, c</sup>

Stock Investment Fund	Code	Average NE (in BRL million)	Accumulated Performance (%)	Stock Investment Fund	Código	PL Médio (R\$ milhões)	Performance Acumulada (%)
ABN AMRO FI ACOES ENERGY	abn1	41,96	39,68	ITAU INST IBOVESPA ATIVO ACOES FI	itau4	37,85	134,16
ATICO ACOES FI EM ACOES	atico	5,82	74,20	ITAU INSTUCIONAL IBRX ATIVO ACOES FI	itau5	26,13	240,52
ATRIUM FIA	atri	2,47	232,80	ITAU PERS MARCHE IBOVESPA ACOES FICFI	itau6	58,35	53,62
FUNDO BANESTES DE INVESTIM. EM ACOES	bane	3,07	65,40	ITAU PERS TECHNIQUE ACOES FI	itau7	35,25	51,73
BANRISUL INDICE FI ACOES	banr1	6,69	65,41	ITAU PERSONNALITE ACOES FICFI	itau8	46,73	102,42
BANRISUL PERFORMANCE FI ACOES	banr2	17,88	149,80	ITAU PRIVATE ATIVO ACOES FI	itau9	21,16	126,14
BB ACOES ENERGIA FI	bb1	26,18	73,61	ITAU PRIVATE EXPERTISE ACOES FICFI	itau10	7,52	241,36
BB ACOES TELECOMUNICACOES FIA	bb2	27,61	-55,27	FI FATOR JAGUAR ACOES	jagu	34,32	216,40
BIC STOCK INDEX FI ACOES	bic1	1,20	2,85	LEGG MASON PORTFOLIO ACOES FI	legg2	95,01	159,91
BNB FI ACOES	bnb	4,31	51,31	MB FUNDO DE INVESTIMENTO EM ACOES	mb	3,76	-23,75
BOREAL ACOES III FIA	boreal	20,24	-23,17	MB FUNDO DE INVESTIMENTO EM ACOES FLEX	mb2	1,56	40,42
BRADESCO FIA BD	brad1	6,19	181,27	MULTI STOCK FUNDO DE INVEST EM ACOES	mult	1,31	-86,57
BRADESCO FIA MULTI SETORIAL	brad2	69,74	13,62	OPPORTUNITY LOGICA II FIA	oppo	1.031,25	536,13
BRADESCO FIA SUPER ACAO	brad3	37,34	23,87	PILLAINVEST FUNDO DE INVEST EM ACOES FI	pill	9,09	-27,05
BRADESCO FIA SEGURIDADE	brad5	11,42	145,57	FUNDO DE INVESTIMENTO EM ACOES PRIME	prim	5,42	135,19
BRB ACOES	brb	5,22	29,64	FUNDO PRIME DE INV EM ACOES CART LIVRE	prim2	0,20	-17,05
CAIXA FI ACOES IBOVESPA	caix	125,39	19,14	PROSPER ADINVEST FIA	pros	6,77	754,49
COINVALORES FIA	coin	5,65	140,46	REAL FI ACOES INSTITUCIONAL	real	26,00	177,37
COMERCIAL MASTER FIA	come	8,09	657,02	REAL FIQ FI ACOES PLUS	real2	15,91	41,21
CA COMPOSITE ACOES FIC FIA	comp	6,23	171,46	SAFRA INDICIAL FI ACOES	safr1	20,92	45,26
CS "FIG" PREMIUM FIA	crsu	35,36	172,23	SAFRA MULTI DIVIDENDOS FI ACOES	safr2	143,17	477,39
CS IBOVESPA INDEX FIA	crsu2	4,81	77,01	SAFRA PRIVATE FI ACOES	safr3	30,56	177,48
DYNAMO COUGAR FIA	dyna	360,19	771,40	SAFRA SETORIAL BANCOS FI ACOES	safr4	107,13	381,49
ELITE FUNDO DE INVESTIMENTO EM ACOES	elit	2,47	181,66	SAFRA SETORIAL ENERGIA FI ACOES	safr5	24,48	104,63
ENERGY IB FIA	ener	17,11	85,20	SANTANDER FI ACOES	sant1	57,19	78,31
FI FATOR ACOES INSTITUCIONAL	fato	37,68	197,12	SANTANDER FI INST ACOES	sant2	36,51	182,38
FIBRA VIC FI ACOES	fibr	2,50	167,47	SLW FIA	slw	2,49	-9,88
GALAXIA ACOES FI	gala	253,73	375,52	SMALL CAP VALUATION IB FIA	smal	75,02	498,48
GAP FI ACOES	gap	42,81	119,00	SUL AMERICA EQUILIBRIUM FIA	sula	13,51	160,41
GERACAO FIA	gera	97,16	404,90	TELECOM IB FIA	tele	22,95	25,55
GRADUAL PAVARINI FIA	grad	18,25	394,63	TEMPO CAPITAL FI ACOES	temp	162,33	1.957,28
HG TOP ACOES FICFIA	hg	43,91	319,63	UNIBANCO BLUE FI ACOES	unib1	112,50	10,59
HSBC FI ACOES INSTITUCIONAL	hsbc1	184,07	169,20	UNIBANCO INSTITUCIONAL IBX FI ACOES	unib2	16,76	184,25
HSBC FIA TOP	hsbc2	59,51	43,73	UNIBANCO PREVIDENCIA IBOVESPA FI ACOES	unib3	22,76	39,12
IP PARTICIPACOES FI ACOES	ip	126,13	356,69	UNIBANCO STRATEGY FI ACOES	unib4	86,35	53,89
ITAU ACOES FI	itau1	444,82	163,82	UNIBANCO TIMING FI ACOES	unib5	11,39	116,20
ITAU CARTEIRA LIVRE ACOES FI	itau2	120,12	29,63	FI VOT ACOES	voto	7,89	71,82
ITAU INDICE ACOES IBOVESPA FICFI	itau3	44,93	42,81				

<sup>a</sup> Panel containing all stock investment funds in activity in Brazil from January 1998 to December 2008, with complete data series (Source: www.fortuna.com.br)

<sup>b</sup> Average NE: arithmetic mean of the monthly time series of the net equity of each investment fund, during the period from 1998.1 to 2008.12, 132 observations

<sup>c</sup> Accumulated Performance: accumulated actual return of each investment fund, during the period from 1998.1 to 2008.12, 132 observations

Table 2 also includes the indicators of accumulated overperformance in absolute terms in relation to the Ibovespa and the respective average values of Net Equity, being possible to observe that from the 75 funds analyzed, 42 had a better performance when compared with the main market portfolio in the country. In view of the trade-off which exists for all econometricians, between longer time series in comparison to a wider cross-section, our goal here was to extend the subject matter of the study and thus the database used in Matos and Rocha (2009), increasing from 18 to 75 investment funds, and updating such base with the inclusion of the years 2007 and 2008, as it is believed that such base is satisfactory, with respect to the intention of deriving results with the use of a modeling with threshold applied for "panels" of funds.<sup>iv</sup>

The time series of the São Paulo Stock Exchange index (Ibovespa) and of the Savings Account, proxy of the risk-free rate, were also extracted from Fortuna.<sup>v</sup>

The savings account, whose remuneration is established by the Reference Rate [Taxa Referencial] (TR) and a fixed percentage of 0.5% capitalized monthly, has an average performance which is lower than the remuneration offered by the SELIC rate. Being such higher remuneration understood as a premium for incurred risk, and the savings account, less risky than government bonds, could be considered as a more appropriate proxy.

Even more relevant than such point is the fact that the volatility of the net return of the savings account can easily be compared to the interests rates in developed economies, at a level of 7% per year, while the volatility of the American government's Treasury Bill is of 3.1% per year and that of SELIC is almost 10% per year, taking into consideration the quarterly data from 1998 to 2007.

With respect to the descriptive statistics of the relevant assets when using the CAPM, the actual average net return per year is 11.80%, for the Ibovespa, and 2.97%, for the savings account, while annual volatilities are approximately 72.53% and 3.10%, respectively. Finally, IBGE's Extended National Consumer Prices Index [Índice de Preço ao Consumidor Amplo] (IPCA) was used, as inflation index, which was, in average, 6.57% per year, with annual volatilities of 2.97%. Before the analysis of the results, we should present the results of the test of the proposed pricing models of the models proposed in Tables 4, 5 and 6 [sic]. Table 3 consists of an auxiliary tool to indicate the relative position of each one of the 75 funds in each of the regression results tables, considering the absolute accumulated overperformance and the average Net Equity. Following Matos and Rocha (2009), this form of grouping was proposed with the intention of facilitating the visualization of evidence or patterns in the pricing capacities of the models in question within the cross-section.

## 4.2. ECONOMETRIC MODELS USED

The benchmark model applied in this study was the CAPM, in accordance with the following equation:

$$R_t^i - R_t^f = a^i + \beta^i \cdot [R_t^m - R_t^f] + \varepsilon_t^i \quad (2)$$

where  $\varepsilon_t$  consists of the regression residual.

The econometric estimation technique used was Ordinary Least Squares (OLS), where, aiming at obtaining robustness to the basic violations of heteroscedasticity and autocorrelation, the correction for the variance-covariance matrix proposed by Newey and West (1987) was adopted, valid particularly for large samples.<sup>vi</sup> As hypothesis test,  $t$  statistics were used to analyze the significance of the *alpha* and *beta* coefficients. The results for the panel of 75 investment funds are listed below in Table 4.

With the purpose of evidencing what is the relevance of the incorporation of a non-linear effect into a traditional modeling of investment fund pricing, having as decision variable the net equity of the respective fund, this article uses an extension of the CAPM model. It is a CAPM model, in which the  $a^i$  and  $\beta^i$  parameters vary in accordance with the level of the variable 'net equity', allowing different correlations between the excess return of the asset and of the market portfolio, where each correlation occurs in what can be defined as regime. Assuming the existence of two regimes, the extension of the CAPM model used, known as CAPM threshold, can be described as follows:

$$R_t^i - R_t^f = \begin{cases} a_1^i + \beta_1^i \cdot [R_t^m - R_t^f] + \varepsilon_{1t}^i, & PL_t^i < \gamma \\ a_2^i + \beta_2^i \cdot [R_t^m - R_t^f] + \varepsilon_{2t}^i, & PL_t^i \geq \gamma \end{cases} \quad (3)$$

where the angular and intercept coefficients have distinct values,  $(a_1^i, \beta_1^i)$  and  $(a_2^i, \beta_2^i)$ , depending on the regime in force, to be identified from the endogenous determination of the threshold value  $\gamma$  for the net equity, which has, throughout time, values higher and lower than the threshold. The residuals are deemed as independent and identically distributed with finite variance within each regime.

The estimation of the parameters of model (3) is made via minimization of the sum of the squared conditional residuals, according to Hansen (1999). For any fixed value of the

threshold  $\gamma$ , the other parameters are estimated by least squares, resulting in the function

$S(\gamma) = \sum \hat{\varepsilon}_t^2(\gamma)$ . The parameter  $\gamma$  is then estimated, finding the value  $\hat{\gamma}$ , such that  $S(\gamma)$  is minimized. Chan (1993) and Hansen (1999) recommend that, in the minimization process, a percentage of the highest and lowest values of the threshold variable observed in the sample are eliminated, thus rejecting the possibility of selecting a value  $\hat{\gamma}$  which has a very high quantity of observations within a single regime, there being, thus, few values in the other regime.<sup>vii</sup> A second and last extension proposed shall be the inclusion of a second threshold, being a scenario which analyzes the possibility of identifying more than two regimes.

The threshold model CAPM with three regimes will have the following structure:

$$R_t^i - R_t^f = \begin{cases} a_1^i + \beta_1^i \cdot [R_t^m - R_t^f] + \varepsilon_{1t}^i, & PL_t^i < \gamma \\ a_2^i + \beta_2^i \cdot [R_t^m - R_t^f] + \varepsilon_{2t}^i, & \gamma \leq PL_t^i < \delta \\ a_3 + \beta_3 \cdot [R_t^m - R_t^f] + \varepsilon_{3t}, & PL_t^i \geq \delta \end{cases} \quad (4)$$

The estimation of the parameters in this model is similar to that of the previous model. For any fixed value of *thresholds*  $\gamma$  and  $\delta$ , the other parameters are estimated by least squares, resulting in the function  $S(\gamma, \delta) = \sum \varepsilon_t^2(\gamma, \delta)$ . The parameters  $\gamma$  and  $\delta$  are then estimated by the minimization of the function  $S(\gamma, \delta)$ . As to the hypothesis tests associated with the three models for each investment fund, initially, it is necessary to evidence whether the model with a single threshold is statistically significant in relation to the model with linear specification, being in such case the null hypothesis ( $H_0$ ) of rejection of the non-linear model. In a second stage, mainly for the investment funds which have rejected the traditional linear modeling, the null hypothesis that the model of rejection of the model with a single threshold in comparison with the model with two thresholds [sic]. Such tests follow the methodological procedures suggested in Hansen (1999). The results of said two non-linear models are contained in Table 5 and 6, respectively.<sup>viii</sup>

## 5. RESULTS

Aiming at providing a better visualization of the identification of a pattern by the investment funds, the layout of Tables 4, 5 and 6 with the results follows the classification reported in Table 3.

**Table 3**  
**Classification of the 75 stock investment funds in Brazil, based on average NE and overperformance**  
**(1998:1 - 2008:12 - 132 observations)<sup>a, b, c, d</sup>**

		<i>Overperformance</i> (ascending order)													
<b>Average NE</b> (ascending order)	low	2	3	4	5	6	7	8	9	10	11	12	13	14	high
	<b>very small</b>	CRSU2	BANE	BNB	MB2	PRIM	BRB	COIN	BIC1	FIBR	SLW	ELIT	PRIM2	MB	ATRI
<b>small</b>	ENER	ATICO	VOTO	BANR1	UNIB5	REAL2	BRAD5	SULA	COMP	BRAD1	UNIB2	PILL	ITAU10	COME	PROS
<b>medium</b>	BB1	SAFR5	ITAU7	SAFR1	ITAU9	UNIB3	TELE	BANR2	REAL	SAFR3	BOREAL	JAGU	BB2	ITAU5	GRAD
<b>large</b>	SANT1	ITAU8	ITAU6	GAP	HSBC2	ITAU3	ABN1	ITAU4	BRAD3	BRAD2	CRSU	SANT2	FATO	HG	SMAL
<b>very large</b>	UNIB4	ITAU2	CAIX	UNIB1	LEGG2	ITAU1	HSBC1	IP	GALA	SAFR4	GERA	SAFR2	OPPO	DYNA	TEMP

<sup>a</sup> Panel containing all stock investment funds in activity in Brazil from January 1998 to December 2008, with complete data series (Source: [www.fortuna.com.br](http://www.fortuna.com.br))

<sup>b</sup> Average NE: arithmetic mean of the monthly time series of the net equity of each investment fund, during the period from 1998:1 to 2008:12, 132 observations.

<sup>c</sup> Overperformance: accumulated actual return of each investment fund in relation to *Ibovespa* (in absolute terms) during the period from January 1998 to December 2008, 132 observations.

<sup>d</sup> The formation of the 5 quintiles is made through the classification in ascending order of the 75 funds with respect to average net equity during the period from January 1998 to December 2009. After that, each quintile is organized in ascending order with respect to overperformance.



**Table 5**  
**Pricing panel of investment funds in Brazil shares with the simple CAPM with Threshold (1998:1 - 2008:12 - 132 observations).**<sup>a,b, c, d</sup>

$$R_t^i - R_t^f = \begin{cases} a_1^i + \beta_1^i \cdot [R_t^m - R_t^f] + \varepsilon_t^i, & PL_t^i < \gamma \\ a_2^i + \beta_2^i \cdot [R_t^m - R_t^f] + \varepsilon_t^i, & PL_t^i \geq \gamma \end{cases}$$

PL médio (ordem crescente)	Overperformance (ascending order)														Overperformance (ascending order)															
	baixa	2	3	4	5	6	7	8	9	10	11	12	13	14	alta	baixa	2	3	4	5	6	7	8	9	10	11	12	13	14	alta
	<b>a<sub>1</sub></b>														<b>b<sub>1</sub></b>															
<b>muito peq.</b>	-0,001	0,005	0,001	0,009	0,004	-0,005	0,030	0,006	-0,006	0,004	0,004	0,024	0,008	0,005	0,049	0,976	0,974	0,994	0,975	0,725	0,824	0,987	0,912	1,011	0,786	0,776	0,769	0,978	0,455	1,062
	(0,0013)*	(0,0028)*	(0,0048)*	(0,0178)*	(0,0046)*	(0,0038)*	(0,0079)	(0,0021)*	(0,0041)*	(0,0037)*	(0,0047)*	(0,0077)*	(0,0067)*	(0,0101)*	(0,0098)*	(0,0130)	(0,0445)	(0,0560)	(0,1542)	(0,0409)	(0,0356)	(0,0503)	(0,0376)	(0,0482)	(0,0386)	(0,0487)	(0,0515)	(0,0496)	(0,0984)	(0,1212)
<b>pequeno</b>	-0,008	0,021	0,001	0,005	0,002	0,003	0,003	0,009	-0,019	0,009	0,004	0,007	0,005	0,032	0,047	1,099	0,641	0,950	1,043	0,613	0,943	0,946	0,731	0,589	0,917	0,851	0,763	0,893	0,943	0,962
	(0,0076)*	(0,0098)*	(0,0037)*	(0,0054)*	(0,0061)*	(0,0020)*	(0,0014)	(0,0068)*	(0,0124)*	(0,0027)*	(0,0020)*	(0,0034)*	(0,0034)*	(0,0167)*	(0,0318)*	(0,0921)	(0,0892)	(0,0645)	(0,0395)	(0,0737)	(0,0362)	(0,0197)	(0,0351)	(0,0797)	(0,0344)	(0,0267)	(0,0316)	(0,0487)	(0,1449)	(0,2837)
<b>médio</b>	0,016	0,011	0,007	0,002	-0,006	0,022	0,028	0,013	0,001	0,013	0,003	0,013	0,014	-0,002	0,008	0,921	0,934	1,185	1,031	1,014	0,780	0,178	0,74305	0,830	1,081	0,81991	0,994	0,85959	1,004	0,780
	(0,0117)*	(0,0084)*	(0,0050)*	(0,0016)*	(0,0029)	(0,0116)*	(0,0167)*	(0,0147)*	(0,0066)*	(0,0063)*	(0,0075)*	(0,0050)	(0,0044)*	(0,0043)*	(0,0032)	(0,1012)	(0,1049)	(0,0482)	(0,0190)	(0,0267)	(0,0797)	(0,2016)	(0,12149)	(0,0315)	(0,0582)	(0,0528)	(0,0585)	(0,0510)	(0,0363)	(0,0375)
<b>grande</b>	0,000	-0,001	-0,001	0,007	0,001	0,000	-0,021	-0,001	0,007	0,010	0,006	0,016	0,004	0,008	0,017	0,940	0,833	1,039	1,099	1,033	1,059	0,919	1,050	0,935	1,021	0,979	0,993	0,968	0,809	0,810
	(0,0023)*	(0,0037)*	(0,0020)*	(0,0092)*	(0,0046)*	(0,0038)*	(0,0069)*	(0,0053)*	(0,0052)*	(0,0024)*	(0,0028)	(0,0054)	(0,0027)*	(0,0032)	(0,0059)	(0,0222)	(0,0350)	(0,0187)	(0,0718)	(0,0373)	(0,0277)	(0,0767)	(0,0406)	(0,0679)	(0,0376)	(0,0257)	(0,0281)	(0,0322)	(0,0397)	(0,0716)
<b>muito gr.</b>	0,006	-0,003	-0,003	0,007	0,002	0,002	0,002	0,019	0,010	0,012	0,010	0,010	0,007	0,019	0,017	0,917	1,046	1,018	0,878	1,023	0,821	0,869	0,756	0,732	0,842	0,819	0,567	0,783	0,535	0,507
	(0,0026)*	(0,0014)*	(0,0013)*	(0,0029)*	(0,0015)*	(0,0036)*	(0,0031)*	(0,0118)*	(0,0037)*	(0,0049)*	(0,0045)	(0,0047)	(0,0196)*	(0,0055)*	(0,0051)	(0,0277)	(0,0185)	(0,0158)	(0,0379)	(0,0185)	(0,0373)	(0,0395)	(0,0712)	(0,0357)	(0,0598)	(0,0490)	(0,0542)	(0,1827)	(0,0578)	(0,0438)
	<b>a<sub>2</sub></b>														<b>b<sub>2</sub></b>															
<b>muito peq.</b>	0,001	0,001	0,001	0,003	0,002	0,001	-0,002	0,003	0,006	0,007	0,001	0,002	-0,006	0,046	0,006	1,015	0,739	0,871	0,838	0,924	0,731	0,978	0,969	0,824	0,976	0,947	0,783	0,825	1,048	0,647
	(0,0015)*	(0,0036)*	(0,0022)*	(0,0022)*	(0,0049)*	(0,0024)*	(0,0025)*	(0,0059)*	(0,0031)*	(0,0037)*	(0,0027)*	(0,0044)*	(0,0025)	(0,0330)*	(0,0080)*	(0,0175)	(0,0468)	(0,0309)	(0,0351)	(0,0832)	(0,0497)	(0,0369)	(0,0720)	(0,0378)	(0,0344)	(0,0380)	(0,0409)	(0,0351)	(0,4100)	(0,0697)
<b>pequeno</b>	0,006	0,003	0,001	0,001	0,000	0,005	0,004	0,004	0,005	0,001	0,002	0,010	0,005	0,008	0,006	0,798	0,882	1,039	0,986	0,988	0,880	0,907	0,961	0,804	0,963	1,068	0,980	0,764	0,792	0,892
	(0,0043)*	(0,0029)*	(0,0018)*	(0,0015)*	(0,0034)*	(0,0076)*	(0,0050)*	(0,0024)*	(0,0021)*	(0,0020)*	(0,0032)*	(0,0022)*	(0,0040)*	(0,0036)*	(0,0036)*	(0,0706)	(0,0398)	(0,0180)	(0,0189)	(0,0518)	(0,0783)	(0,0407)	(0,0347)	(0,0255)	(0,0234)	(0,0421)	(0,0404)	(0,0627)	(0,0496)	(0,0423)
<b>médio</b>	0,007	0,009	0,000	0,001	0,004	0,001	0,016	0,005	0,004	0,001	0,011	0,005	0,001	0,007	0,006	0,745	0,753	0,994	0,940	1,005	0,950	0,085	1,019	0,991	0,931	0,524	0,955	1,205	0,909	1,040
	(0,0042)*	(0,0047)*	(0,0019)*	(0,0013)*	(0,0016)*	(0,0017)*	(0,0101)*	(0,0049)*	(0,0017)*	(0,0036)*	(0,0062)*	(0,0031)*	(0,0073)*	(0,0021)*	(0,0069)*	(0,0627)	(0,0579)	(0,0284)	(0,0121)	(0,0270)	(0,0193)	(0,1205)	(0,0809)	(0,0258)	(0,0473)	(0,0795)	(0,0566)	(0,0782)	(0,0259)	(0,0767)



	-0,001	0,003	-0,001	0,001	-0,002	-0,002	0,006	0,004	0,000	0,002	-0,003	0,001	0,005	0,004	0,001	1,031	1,003	0,967	0,994	0,973	0,980	0,799	0,977	1,074	0,913	1,030	0,970	0,859	0,969	1,009
<b>grande</b>	(0,0018)*	(0,0024)*	(0,0011)*	(0,0021)*	(0,0009)	(0,0009)*	(0,0040)*	(0,0014)	(0,0026)*	(0,0026)*	(0,0045)*	(0,0014)*	(0,0040)*	(0,0041)*	(0,0046)*	(0,0274)	(0,0355)	(0,0125)	(0,0222)	(0,0111)	(0,0109)	(0,0584)	(0,0162)	(0,0528)	(0,0284)	(0,0427)	(0,0183)	(0,0631)	(0,0463)	(0,0705)
	0,001	-0,001	-0,003	0,002	0,007	0,002	0,004	0,005	0,002	0,022	0,005	0,006	0,010	0,003	0,020	0,970	0,908	0,958	0,993	1,080	1,014	1,044	0,570	1,041	1,343	1,263	0,935	1,017	0,754	0,788
<b>muito gr.</b>	(0,0020)*	(0,0042)*	(0,0017)*	(0,0015)*	(0,0046)*	(0,0024)*	(0,0025)*	(0,0036)*	(0,0029)*	(0,0078)*	(0,0064)*	(0,0042)*	(0,0040)*	(0,0038)*	(11,1519)	(0,0267)	(0,0417)	(0,0206)	(0,0201)	(0,0448)	(0,0348)	(0,0370)	(0,0487)	(0,0375)	(0,1022)	(0,0909)	(0,0716)	(0,0789)	(0,0557)	(0,0411)

	<b>R<sup>2</sup></b>															<b>LR Test</b>														
<b>muito peq.</b>	0,982	0,898	0,919	0,807	0,788	0,918	0,891	0,921	0,884	0,880	0,873	0,730	0,901	-0,752	0,238	4,5029	7,168	5,092	4,474	4,678	18,508	4,247	9,418	12,152	7,868	3,693	8,474	5,886	12,532	
<b>pequeno</b>	0,713	0,852	0,931	0,961	0,815	0,914	0,961	0,908	0,892	0,953	0,947	0,921	0,853	0,722	0,223	14,423	17,962	4,592	5,741	25,938	2,367	4,486	24,774	23,811	5,755	13,875	16,378	2,785	8,034	6,252
<b>médio</b>	0,652	0,637	0,941	0,984	0,961	0,917	-0,329	0,572	0,927	0,823	0,564	0,875	0,754	0,934	0,849	8,108	7,100	20,175	20,179	8,927	25,441	5,701	6,706	15,399	9,876	9,625	5,499	19,892	8,802	9,220
<b>grande</b>	0,961	0,942	0,985	0,915	0,980	0,984	0,787	0,967	0,868	0,946	0,896	0,962	0,890	0,891	0,751	6,879	16,399	13,341	5,453	7,859	14,277	12,639	8,381	7,315	17,166	4,033	13,913	4,228	6,271	8,371
<b>muito gr.</b>	0,951	0,967	0,980	0,963	0,958	0,906	0,928	0,755	0,873	0,668	0,755	0,817	0,634	0,763	0,799	----	13,852	----	17,059	----	12,137	13,097	----	22,944	10,151	18,649	21,596	----	9,393	12,514
																[0,125]**	[0,932]**	[0,260]**	[0,296]**	[0,346]**	[0,029]	[0,193]**	[0,387]**	[0,875]**	[0,917]**	[0,977]**	[0,156]**	[0,674]**	[0,477]**	[0,877]**

**Threshold (in R\$ millions)**

<b>muito peq.</b>	R\$ 4,97	R\$ 2,90	R\$ 2,75	R\$ 0,23	R\$ 6,35	R\$ 2,41	R\$ 2,14	R\$ 1,60	R\$ 1,65	R\$ 1,93	R\$ 1,31	R\$ 0,11	R\$ 2,72	R\$ 3,98	R\$ 1,00
<b>pequeno</b>	R\$ 16,39	R\$ 2,20	R\$ 3,10	R\$ 2,23	R\$ 6,86	R\$ 27,27	R\$ 22,90	R\$ 3,42	R\$ 2,41	R\$ 1,79	R\$ 21,39	R\$ 8,64	R\$ 11,56	R\$ 2,01	R\$ 1,91
<b>médio</b>	R\$ 11,98	R\$ 5,55	R\$ 10,63	R\$ 18,84	R\$ 11,30	R\$ 2,79	R\$ 5,32	R\$ 4,22	R\$ 9,37	R\$ 13,12	R\$ 22,03	R\$ 4,89	R\$ 31,52	R\$ 11,78	R\$ 32,55
<b>grande</b>	R\$ 38,25	R\$ 39,68	R\$ 31,36	R\$ 1,44	R\$ 23,04	R\$ 22,60	R\$ 4,65	R\$ 26,60	R\$ 29,13	R\$ 58,77	R\$ 31,47	R\$ 6,79	R\$ 38,50	R\$ 16,63	R\$ 60,92
<b>muito gr.</b>	R\$ 77,53	R\$ 168,52	R\$ 124,98	R\$ 95,86	R\$ 129,00	R\$ 399,05	R\$ 96,46	R\$ 85,89	R\$ 144,47	R\$ 302,70	R\$ 60,41	R\$ 106,31	R\$ 218,24	R\$ 466,42	R\$ 53,52

\* Parâmetro não significativo ao nível de 5%. Erro padrão entre parênteses. \*\* Não rejeita a hipótese nula de que o modelo restrito seja o mais adequado ao nível de 5%. P-valor Bootstrap entre entre colchetes.

<sup>a</sup> Técnica de estimação: Mínimos Quadrados Ordinários com correção para heterocedasticidade. P-valor (Bootstrap) obtido a partir de 1000 replicações segundo técnica de Hansen (1999).

<sup>b</sup> Painel contendo todos os fundos de investimento em ações em atividade no Brasil e com série temporal completa, durante o período de janeiro de 1998 a dezembro de 2008 (Fonte: www.fortuna.com.br)

<sup>c</sup> PL médio: média aritmética da série temporal mensal do patrimônio líquido de cada fundo de investimento, durante o período de janeiro de 1998 a dezembro de 2008, 132 observações.

<sup>d</sup> Overperformance: retorno real acumulado excedente em relação ao Ibovespa (em termos absolutos) de cada fundo de investimento, durante o período de janeiro de 1998 a dezembro de 2008, 132 observações.

**Table 6**  
**Pricing panel of investment funds in Brazil shares with the CAPM model with double Threshold (1998:1 - 2008:12 - 132 observations).**<sup>a,b,c,d</sup>

$$R_t^i - R_{t,f}^i = \begin{cases} a_1^i + \beta_1^i \cdot [R_t^m - R_{t,f}^f] + \varepsilon_t^i, & PL_t^i < \gamma \\ a_2^i + \beta_2^i \cdot [R_t^m - R_{t,f}^f] + \varepsilon_t^i, & \gamma \leq PL_t^i < \delta \\ a_3^i + \beta_3^i \cdot [R_t^i - R_{t,f}^i] + \varepsilon_t^i, & PL_t^i \geq \delta \end{cases}$$

PL médio (ordem crescente)	Overperformance (ascending order)														Overperformance (ascending order)															
	baixa	2	3	4	5	6	7	8	9	10	11	12	13	14	alta	baixa	2	3	4	5	6	7	8	9	10	11	12	13	14	alta
	<b>a<sub>1</sub></b>														<b>b<sub>1</sub></b>															
<b>muito peq.</b>	0,000 (0,001 6)*	0,005 (0,002 8)*	0,001 (0,004 8)*	0,009 (0,017 8)*	0,004 (0,004 6)*	0,005 (0,003 8)*	0,030 (0,007 9)	0,004 (0,002 8)*	0,006 (0,004 1)*	0,001 (0,0042)*	0,003 (0,006 7)*	-0,024 (0,0077 )*	-0,008 (0,006 7)*	0,021 (0,022 4)*	-0,049 (0,009 8)*	0,979 (0,029 2)	0,974 (0,044 5)	0,994 (0,056 0)	0,975 (0,154 2)	0,725 (0,040 9)	0,824 (0,035 6)	0,987 (0,050 3)	0,983 (0,041 7)	1,011 (0,048 2)	0,797 (0,049 0)	0,915 (0,037 1)	0,769 (0,051 5)	0,978 (0,049 6)	0,725 (0,232 1)	1,062 (0,121 2)
<b>pequeno</b>	0,022 (0,010 1)	0,021 (0,009 7)*	0,006 (0,003 4)*	0,005 (0,005 4)*	0,011 (0,007 2)*	-0,002 (0,002 1)*	0,001 (0,001 4)*	0,009 (0,006 8)*	0,019 (0,012 4)*	0,009 (0,0027)	0,005 (0,004 6)*	-0,016 (0,0065 )*	0,013 (0,013 0)*	0,032 (0,016 7)*	0,047 (0,031 8)*	0,994 (0,089 9)	0,641 (0,089 2)	1,028 (0,021 3)	1,043 (0,039 5)	0,681 (0,081 8)	0,909 (0,044 4)	0,933 (0,019 7)	0,731 (0,035 1)	0,589 (0,079 7)	0,917 (0,034 4)	0,863 (0,043 3)	0,760 (0,036 3)	0,876 (0,096 5)	0,943 (0,144 9)	0,962 (0,283 7)
<b>médio</b>	0,016 (0,016 8)*	0,011 (0,008 4)*	0,007 (0,005 0)*	-0,002 (0,001 6)*	-0,006 (0,002 9)	0,022 (0,011 6)*	0,028 (0,016 7)*	0,013 (0,014 7)*	-0,001 (0,006 6)*	0,013 (0,0063)	0,000 (0,007 5)*	0,013 (0,0051 )	0,009 (0,006 6)*	-0,002 (0,004 3)*	0,012 (0,004 0)	0,921 (0,101 2)	0,934 (0,104 9)	1,185 (0,048 2)	1,031 (0,019 0)	1,014 (0,026 7)	0,780 (0,079 7)	0,178 (0,201 6)	0,743 (0,121 4)	0,830 (0,031 5)	1,081 (0,058 2)	0,820 (0,052 8)	0,994 (0,058 5)	0,695 (0,075 4)	1,004 (0,036 3)	0,781 (0,039 9)
<b>grande</b>	-0,002 (0,004 6)*	-0,005 (0,004 6)*	-0,001 (0,002 0)*	0,007 (0,009 2)*	0,001 (0,004 6)*	0,000 (0,003 8)*	0,210 (0,006 9)*	-0,001 (0,005 3)*	0,007 (0,005 2)*	0,005 (0,0022)*	0,002 (0,003 2)*	0,016 (0,0055 )	0,003 (0,003 4)*	0,001 (0,007 0)*	0,013 (0,007 6)*	0,987 (0,024 6)	0,938 (0,025 0)	1,039 (0,018 7)	1,099 (0,071 8)	1,033 (0,037 3)	1,059 (0,027 7)	0,919 (0,076 7)	1,050 (0,040 6)	0,935 (0,067 9)	0,969 (0,039 5)	0,959 (0,027 0)	0,993 (0,028 1)	0,904 (0,039 5)	0,879 (0,057 7)	0,912 (0,069 2)
<b>muito gr.</b>	0,006 (0,002 6)*	-0,004 (0,001 3)	0,004 (0,001 4)*	0,007 (0,002 9)*	0,006 (0,002 0)	0,005 (0,009 7)*	0,000 (0,004 8)*	0,019 (0,011 8)*	0,022 (0,006 4)	0,001 (0,0103)*	-0,018 (0,012 7)*	0,005 (0,0086 )*	0,007 (0,019 6)*	0,018 (0,007 7)	0,020 (0,007 5)	0,917 (0,027 7)	1,023 (0,017 1)	1,006 (0,014 4)	0,878 (0,037 9)	1,035 (0,020 0)	0,899 (0,064 4)	0,915 (0,056 7)	0,756 (0,071 2)	0,820 (0,051 3)	0,869 (0,093 2)	0,829 (0,109 4)	0,506 (0,081 7)	0,783 (0,182 7)	0,610 (0,057 4)	0,569 (0,052 1)
	<b>a<sub>2</sub></b>														<b>b<sub>2</sub></b>															
<b>muito peq.</b>	0,005 (0,002 5)*	0,001 (0,003 7)*	0,002 (0,002 7)*	-0,010 (0,004 4)*	0,002 (0,006 9)*	0,000 (0,005 3)*	0,002 (0,003 0)*	0,008 (0,002 9)*	0,002 (0,008 5)*	0,015 (0,0065)*	0,007 (0,005 4)*	0,003 (0,0065 3)*	0,008 (0,002 7)*	0,009 (0,009 9)*	-0,024 (0,011 0)*	0,964 (0,015 6)	0,619 (0,039 9)	0,811 (0,032 3)	0,842 (0,060 7)	1,202 (0,120 7)	0,609 (0,085 3)	0,960 (0,042 7)	0,837 (0,036 4)	0,720 (0,064 1)	0,769 (0,059 8)	0,623 (0,051 7)	0,733 (0,046 4)	0,812 (0,036 8)	0,331 (0,084 2)	0,647 (0,081 5)
<b>pequeno</b>	0,001 (0,009 5)*	0,004 (0,003 3)*	0,003 (0,008 5)*	0,005 (0,001 7)*	0,008 (0,010 1)*	-0,003 (0,004 6)*	0,011 (0,004 0)	0,015 (0,003 6)	0,008 (0,002 3)	0,002 (0,0025)*	0,007 (0,002 2)	-0,003 (0,0039 )	0,002 (0,002 2)*	0,008 (0,004 6)*	0,009 (0,005 1)	1,222 (0,164 9)	0,860 (0,040 9)	0,794 (0,187 6)	0,999 (0,026 1)	0,488 (0,142 8)	1,078 (0,048 6)	1,001 (0,045 0)	0,977 (0,047 2)	0,795 (0,026 0)	0,954 (0,027 2)	0,851 (0,023 6)	0,762 (0,061 6)	0,906 (0,033 9)	0,689 (0,056 8)	0,769 (0,051 0)
<b>médio</b>	0,009 (0,0047 )*	0,023 (0,011 1)*	-0,005 (0,002 8)*	-0,002 (0,001 3)*	0,009 (0,002 8)	0,006 (0,004 8)*	0,045 (0,017 5)*	0,013 (0,009 6)*	0,007 (0,005 3)	0,003 (0,0043)*	0,007 (0,005 1)*	0,004 (0,0036 )*	0,022 (0,005 )	0,010 (0,002 3)	-0,005 (0,002 6)*	0,786 (0,070 5)	0,860 (0,112 4)	1,002 (0,052 8)	0,958 (0,017 8)	1,018 (0,042 2)	1,002 (0,038 9)	0,238 (0,130 5)	1,095 (0,123 5)	1,125 (0,084 3)	0,813 (0,056 0)	0,315 (0,069 9)	0,884 (0,062 5)	0,966 (0,042 9)	0,852 (0,023 3)	0,789 (0,046 1)

	0,002 (0,002 3)*	0,001 (0,002 6)*	0,000 (0,001 2)*	0,005 (0,003 3)*	-0,004 (0,002 6)*	-0,001 (0,001 0)*	0,002 (0,006 6)*	0,002 (0,001 7)*	0,005 (0,004 9)*	0,014 (0,0039)*	0,013 (0,005 9)*	0,002 (0,0019 )*	0,008 (0,004 2)*	0,011 (0,003 3)*	0,024 (0,005 0)	0,876 (0,027 3)	0,736 (0,029 1)	0,947 (0,014 4)	0,971 (0,032 7)	0,949 (0,024 2)	0,972 (0,011 5)	0,598 (0,085 1)	0,980 (0,020 2)	1,234 (0,085 0)	1,044 (0,039 9)	0,981 (0,059 4)	0,954 (0,022 6)	1,041 (0,027 1)	0,753 (0,030 6)	0,438 (0,047 8)	
<b>grande</b>	0,004 (0,002 5)*	0,003 (0,004 0)*	0,001 (0,002 3)*	0,002 (0,003 6)*	-0,003 (0,002 2)*	0,000 (0,002 5)*	0,000 (0,003 6)*	0,004 (0,010 6)*	0,010 (0,006 6)*	0,006 (0,004 2)*	0,018 (0,0053)	0,017 (0,004 3)*	0,015 (0,0047 )	0,015 (0,013 4)	0,023 (0,005 9)	0,016 (0,005 1)	1,017 (0,023 1)	1,141 (0,043 8)	1,045 (0,035 6)	1,031 (0,038 7)	1,014 (0,031 6)	0,740 (0,026 8)	0,827 (0,036 8)	0,435 (0,066 2)	0,654 (0,051 1)	0,800 (0,073 0)	0,797 (0,054 7)	0,665 (0,058 4)	1,173 (0,190 2)	0,281 (0,087 2)	0,319 (0,043 1)
	<b>a<sub>3</sub></b>															<b>b<sub>3</sub></b>															
<b> muito peq.</b>	0,001 (0,001 5)*	0,006 (0,004 8)*	0,001 (0,002 7)*	0,001 (0,002 5)*	-0,003 (0,006 2)*	0,000 (0,002 4)*	0,002 (0,004 2)*	0,003 (0,005 9)*	0,007 (0,003 2)*	0,007 (0,0037)*	0,001 (0,002 7)	-0,007 (0,0055 7)*	0,005 (0,006 2)*	0,046 (0,033 0)*	0,011 (0,011 1)*	1,015 (0,017 5)	1,012 (0,047 6)	1,030 (0,033 1)	0,841 (0,037 1)	0,819 (0,080 6)	0,810 (0,042 4)	1,084 (0,038 3)	0,969 (0,072 0)	0,879 (0,039 2)	0,976 (0,034 4)	0,947 (0,038 0)	0,915 (0,074 6)	0,883 (0,091 0)	1,048 (0,410 0)	0,623 (0,108 0)	
<b>pequeno</b>	0,006 (0,004 3)*	0,003 (0,005 4)*	0,001 (0,001 8)*	0,002 (0,002 4)*	0,000 (0,003 4)*	0,005 (0,007 6)	0,004 (0,005 0)*	0,007 (0,002 6)*	0,004 (0,005 0)*	0,006 (0,0031)*	0,002 (0,003 2)*	-0,010 (0,022 )*	0,005 (0,004 0)*	0,005 (0,004 7)*	0,003 (0,004 6)*	0,798 (0,070 6)	1,053 (0,082 8)	1,039 (0,018 0)	0,982 (0,026 7)	0,988 (0,051 8)	0,880 (0,078 3)	0,907 (0,040 7)	0,981 (0,025 5)	0,850 (0,078 1)	1,007 (0,045 0)	1,068 (0,042 1)	0,980 (0,040 4)	0,764 (0,062 7)	1,080 (0,081 8)	1,066 (0,066 2)	
<b>médio</b>	0,002 (0,008 5)*	0,002 (0,004 8)*	0,004 (0,002 4)*	0,002 (0,002 6)*	0,002 (0,002 0)*	0,003 (0,001 5)*	0,003 (0,011 7)*	0,002 (0,004 0)*	0,003 (0,001 7)*	-0,004 (0,0054)*	0,013 (0,010 6)*	-0,002 (0,0050 )*	0,001 (0,007 3)*	0,002 (0,003 2)*	0,006 (0,006 9)*	0,525 (0,114 1)	0,701 (0,069 3)	0,993 (0,025 6)	0,918 (0,016 9)	0,989 (0,025 6)	0,927 (0,019 2)	0,048 (0,140 0)	0,897 (0,060 3)	0,964 (0,020 6)	1,035 (0,065 7)	0,819 (0,117 6)	1,098 (0,061 5)	1,205 (0,078 2)	1,067 (0,041 2)	1,040 (0,076 7)	
<b>grande</b>	-0,001 (0,001 8)*	0,003 (0,002 4)*	-0,001 (0,002 2)*	0,002 (0,002 6)*	-0,002 (0,001 0)*	-0,002 (0,002 5)*	0,008 (0,004 7)*	0,008 (0,002 3)	0,003 (0,002 9)*	0,002 (0,0026)*	0,003 (0,004 5)*	0,002 (0,0018 )*	0,005 (0,004 1)*	0,004 (0,004 1)*	0,001 (0,004 6)*	1,031 (0,027 4)	1,003 (0,035 5)	1,021 (0,022 8)	1,017 (0,030 8)	0,982 (0,011 9)	1,029 (0,034 0)	0,937 (0,067 8)	0,962 (0,023 3)	1,004 (0,036 2)	0,913 (0,028 4)	1,030 (0,042 7)	1,030 (0,025 7)	0,859 (0,063 1)	0,969 (0,046 3)	1,009 (0,070 5)	
<b> muito gr.</b>	0,002 (0,003 1)*	-0,001 (0,004 2)*	0,003 (0,001 7)*	0,004 (0,001 6)*	0,007 (0,004 6)*	0,002 (0,002 4)*	0,004 (0,002 5)*	0,001 (0,003 2)*	0,002 (0,002 9)*	-0,022 (0,0078)*	0,005 (0,006 4)*	0,006 (0,0042 )*	0,009 (0,003 8)	0,003 (0,003 8)*	0,020 (0,004 3)	0,933 (0,038 9)	0,908 (0,041 7)	0,958 (0,020 6)	0,982 (0,022 3)	1,080 (0,044 8)	1,014 (0,034 8)	1,044 (0,036 8)	0,694 (0,045 8)	1,041 (0,037 5)	1,343 (0,102 2)	1,263 (0,090 9)	0,935 (0,071 6)	0,942 (0,060 8)	0,754 (0,055 7)	0,788 (0,041 1)	
	<b>R<sup>2</sup></b>															<b>LR Test</b>															
<b> muito peq.</b>	0,982	0,917	0,926	0,810	0,795	0,923	0,893	0,926	0,891	0,885	0,892	0,738	0,905	-0,687	0,270	2,284	31,180	12,884	1,867	4,549	8,606	2,729	9,061	8,115	5,071	22,78 7	3,658	5,841	5,086	5,807	
<b>pequeno</b>	0,729	0,856	0,939	0,962	0,827	0,919	0,963	0,921	0,894	0,955	0,950	0,924	0,855	0,746	0,238	7,639	3,584	17,128	4,861	9,749	7,010	7,714	22,583	3,281	5,370	8,097	4,686	2,016	12,602	2,541	
<b>médio</b>	0,660	0,648	0,943	0,984	0,962	0,919	-0,276	0,588	0,929	0,833	0,598	0,880	0,772	0,942	0,854	2,918	4,076	5,524	3,722	4,879	3,693	5,483	5,021	4,779	7,765	11,29 9	6,190	10,712	18,289	4,981	
<b>grande</b>	0,964	0,952	0,985	0,916	0,981	0,984	0,792	0,968	0,875	0,947	0,899	0,963	0,893	0,896	0,786	1	26,111	6,096	2,471	2,139	2,382	6	2,417	7,436	4,071	3,537	3,955	3,880	6,125	21,18 2	
<b> muito gr.</b>	0,954	0,970	0,981	0,964	0,960	0,912	0,930	0,767	0,885	0,676	0,773	0,824	0,645	0,781	0,810	7,374	14,940	3,542	3,037	7,178	9,315	3,765	6,491	13,840	3,346	10,22 6	5,856	4,111	10,710	7,431	
																[0,837 **]		[0,631 **]	[0,397 **]	[0,001 **]	[0,182 **]	[0,595 **]	[0,084 **]	[0,074 **]	[0,056 **]	[0,161 **]	[0,691 **]	[0,512 **]	[0,324 **]	[0,049 ]	
	<b>Threshold 1 (in R\$ millions)</b>															<b>Threshold 2 (in R\$ millions)</b>															
<b> muito pequeno</b>	R\$ 3,41	R\$ 2,90	R\$ 2,75	R\$ 0,23	R\$ 6,35	R\$ 2,41	R\$ 2,14	R\$ 0,90	R\$ 1,65	R\$ 1,10	R\$ 0,70	R\$ 0,11	R\$ 2,72	R\$ 1,61	R\$ 1,00	R\$ 4,97	R\$ 4,56	R\$ 4,56	R\$ 1,18	R\$ 7,58	R\$ 4,86	R\$ 7,92	R\$ 1,60	R\$ 2,06	R\$ 1,93	R\$ 1,31	R\$ 0,23	R\$ 4,45	R\$ 3,98	R\$ 1,41	
<b>pequeno</b>	R\$ 6,53	R\$ 2,20	R\$ 1,86	R\$ 2,23	R\$ 3,36	R\$ 17,28	R\$ 15,03	R\$ 3,42	R\$ 2,41	R\$ 1,79	R\$ 8,46	R\$ 6,11	R\$ 3,57	R\$ 2,01	R\$ 1,91	R\$ 16,39	R\$ 11,45	R\$ 3,10	R\$ 6,53	R\$ 6,86	R\$ 27,27	R\$ 22,90	R\$ 11,13	R\$ 13,01	R\$ 9,57	R\$ 21,39	R\$ 8,64	R\$ 11,56	R\$ 9,26	R\$ 5,17	
<b>médio</b>	R\$ 11,98	R\$ 5,55	R\$ 10,63	R\$ 18,84	R\$ 11,30	R\$ 2,79	R\$ 5,32	R\$ 4,22	R\$ 9,37	R\$ 13,12	R\$ 22,03	R\$ 4,89	R\$ 22,70	R\$ 11,78	R\$ 19,61	R\$ 43,41	R\$ 7,89	R\$ 17,28	R\$ 24,80	R\$ 20,33	R\$ 12,95	R\$ 12,22	R\$ 11,29	R\$ 20,51	R\$ 31,19	R\$ 31,70	R\$ 71,49	R\$ 31,52	R\$ 32,23	R\$ 32,55	
<b>grande</b>	R\$ 21,69	R\$ 9,40	R\$ 31,36	R\$ 1,44	R\$ 23,04	R\$ 22,60	R\$ 4,65	R\$ 26,60	R\$ 29,13	R\$ 14,65	R\$ 21,14	R\$ 6,79	R\$ 33,06	R\$ 2,41	R\$ 27,84	R\$ 38,25	R\$ 39,68	R\$ 71,77	R\$ 12,46	R\$ 38,39	R\$ 67,49	R\$ 8,68	R\$ 47,63	R\$ 34,85	R\$ 58,77	R\$ 31,47	R\$ 48,15	R\$ 38,50	R\$ 16,63	R\$ 60,92	
<b> muito gr.</b>	R\$ 77,54	R\$ 28,77	R\$ 109,49	R\$ 95,86	R\$ 82,88	R\$ 25,16	R\$ 36,18	R\$ 85,89	R\$ 81,96	R\$ 10,64	R\$ 2,75	R\$ 18,24	R\$ 122,70	R\$ 14,88	R\$ 14,88	R\$ 89,32	R\$ 168,52	R\$ 124,98	R\$ 109,66	R\$ 129,00	R\$ 399,05	R\$ 96,46	R\$ 112,24	R\$ 144,47	R\$ 302,70	R\$ 60,41	R\$ 106,31	R\$ 490,34	R\$ 466,42	R\$ 53,52	

<sup>a</sup> Técnica de estimação: Mínimos Quadrados Ordinários com correção para heterocedasticidade. P-valor (Bootstrap) obtido a partir de 1000 replicações segundo técnica de Hansen (1999).  
<sup>b</sup> Painel contendo todos os fundos de investimento em ações em atividade no Brasil e com série temporal completa, durante o período de janeiro de 1998 a dezembro de 2008 (Fonte: www.fortuna.com.br)  
<sup>c</sup> PL médio: média aritmética da série temporal mensal do patrimônio líquido de cada fundo de investimento, durante o período de janeiro de 1998 a dezembro de 2008, 132 observações.  
<sup>d</sup> Overperformance: retorno real acumulado excedente em relação ao Ibovespa (em termos absolutos) de cada fundo de investimento, durante o período de janeiro de 1998 a dezembro de 2008, 132 observações.

The classification presented in Table 3 follows Fama and French (1993), but taking into consideration the absolute outperformance and the net equity of the funds, instead of the size and value of stocks effects. Thus, aiming at identifying patterns in NE and performance, the classification is such that each one of the five divisions of average net equity is subdivided into 15 subdivisions formed based on the accumulated performance.

The intention of estimating the CAPM, as well as non-linear extensions, consists of accommodating the arithmetic of Sharpe (1991) for funds, so that a framework is more efficient, in case the estimation of Jensen's  $\alpha^i$  parameter, i.e., the intercept, is not significant. Such intercept can also be interpreted as a type of significant average effect of the other omitted variables capable of explaining excess returns of the stock investment funds.

Thus, the results reported in Table 4 obtained from the CAPM, the benchmark model, are analyzed. The exercise is validated, since the parameter associated with the only explanatory variable has statistical significance at 5%, for all the 75 investment funds analyzed, being possible to observe that the values for such measures of sensitivity to the market risk oscillate between 0.51 and 1.18 approximately, with an average slightly lower than 1. The coefficients of determination  $R^2$  of all the funds in question had values with considerable order of magnitude for a model with only one risk factor, exceeding 0.43, indicating the robustness of the relevance of the market excess return with explanatory variable. The only exception was the fund Tele COM IB FIA (TELE), which had a practically null  $R^2$ .

The analysis of the intercept is more relevant. Obviously, stock investment funds which follow passively such market index are modeled satisfactorily by the CAPM. Therefore, it is proven intuitively that the accumulated outperformance seems to clearly influence in the pricing capacity of the CAPM, to the effect of efficiency of the framework in implying non-significant alphas. Such effect is reinforced with the consideration of the net equity as well, in view of the significance at 5% of Jensen's  $\alpha^i$  parameter. More precisely, for the 75 investment funds analyzed, 26 had significant alpha, with the majority having positive values for said performance parameter adjusted for risk. In terms of pattern, a concentration of such significance is observed, in the right and lower region of Table 4, i.e., funds with active strategy with high levels of gain and size with high order of magnitude. This preliminary result corroborates the evidence obtained previously in Matos and Rocha (2009), for a panel of 18 stock investment funds in Brazil.

In short, the result suggests the incapacity on the part of CAPM in capturing the common sources of risk among the major stock investment funds successful in the adoption of active strategy of dynamic rebalancing. Would an improvement in modeling performance be possible by incorporating new factors associated directly or indirectly with such accounting and financial variables?

This is certainly not a trivial question. As the authors Fama and French (1993) themselves argued, even if such factor models were used to price the Fama-French portfolios in an in-sample exercise (within the sample), a good performance could not be guaranteed. This article, however, analyzes this question, but in an indirect manner, by incorporating a

non-linear effect to the traditional pricing modeling, similar to the works of Akdeniz et al. (2003), but having as decision variable the net equity of the respective fund.

Tables 5 and 6 present the results with one and two *thresholds*, respectively. With respect to the explanatory capacity, measured by  $R^2$ , it is possible to detect a subtle worsening of the value of such indicator when using the non-linear CAPM, compared with the benchmark model. Although the average value remains the same for such indicator, for the majority of the funds, the explanation oscillates between 0.20 and 0.98.

In general, concerning thresholds, it is perceived that when only one change of regime is inserted, the thresholds oscillate between 100,000 and 400,000,000 Reais, while with the insertion of two breaks, such values oscillate between 100,000 and 220,000,000 Reais, for the first break, and from 230,000,000 to 490,000,000 Reais, for the second break.

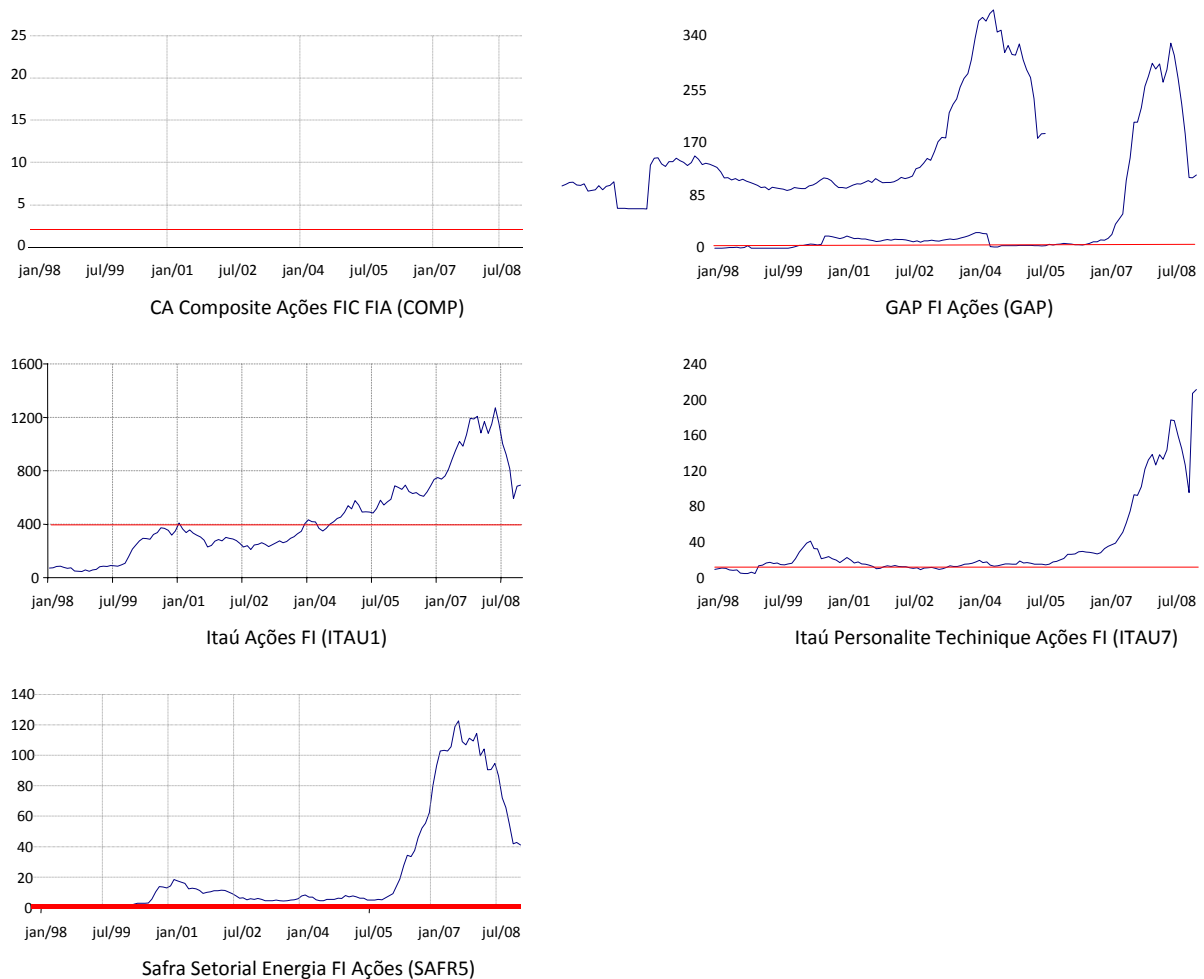
With the exception of the fund Tele COM IB FIA (TELE), all funds had coefficients of determination  $R^2$  with values of considerable order of magnitude for a model with only one risk factor, exceeding 0.43, indicating the robustness of the relevance of the market excess return with explanatory variable.

With respect to the significance of the coefficients for the models with two and three regimes, a robustness of the relevance of the market risk premium is evidenced, in view of the orders of such coefficient and its significance for each regime, validating the exercise. For two NE regimes, an increase of the systemic risk measured by the beta is perceived for the majority of the funds analyzed. With three regimes, the beta reduces from the regime with lower NE to the intermediate regime, having, after that, a strong increase of the values of such risk in the third regime. In all specifications, there does not seem to be a significant increase of the dispersion of the values of sensitivity to market risk. In both exercises, there is evidence that the systemic risk for a same fund increases when it starts to have very high levels of NE.

Observing results listed for the estimations of the alpha coefficients in Tables 4, 5 and 6, it is possible to evidence that the incorporation of breaks reduces by almost 50% the quantity of significant alphas, mainly for regimes characterized by higher net equity. That is, the incorporation of regime changes aggregates by generating non-significant Jensen's alphas, allowing an efficiency gain of the pricing framework, even when not advised vis-a-vis the original model based on the statistical test. Specifically, except for the investment fund CA Composite Ações FIC FIA (COMP), the null hypothesis of adoption of the traditional CAPM is rejected. The use of the unrestricted modeling with only one threshold is advisable for a small sample of funds with a pattern characterized by high net equity and low outperformance.

For the investment funds *Itaú Personalite Technique Ações FI* (ITAU7), *GAP FI Ações* (GAP), *Itaú Ações FI* (ITAU1) and *CA Composite Ações FIC FIA* (COMP) the model with only one threshold showed to be indicated, so that the values of the threshold are significantly high when the respective time series of NE are observed. As to the analysis of the modeling with two thresholds, only for the investment fund *Safra Setorial Energia FI Ações* (SAFR5), the unrestricted model, which now has three regimes seems to be better specified than that with a single threshold, with the null hypothesis being rejected only at 10% significance.

Concerning the thresholds of said five funds, except for the fund *Itaú Ações FI* (ITAU1), which has a NE level considerably higher than almost all the other funds, they assume values between 1 and 10 million Reais, considerably lower than the average NE of the respective funds. Another aspect is the fact that, while for *Itaú Ações FI* (ITAU1) the first regime comprises the period between 1998 and 2004, for the other four funds such period extends to 2005, being a common characteristic of all funds in question the fact that such funds, for which the incorporation of break is shown to be more adequate, are among those which had higher growth, higher NE volatility between 2004 and 2007, with the two regimes endogenously chosen being quite different with respect to this accounting variable, as can be observed in Figure 1.



**FIGURE 1: EVOLUTION OF THE NET EQUITY OF THE INVESTMENT FUNDS COMP, GAP, ITAU1, ITAU7 AND SAFR5**

While Chen et al. (2004) evidence that there is a negative effect of size in the performance of funds, which could be associated with organization or liquidity, for these 5 funds there is evidence of the size effect in the nature of the time series of returns, which starts to have greater systemic risk, to the point of it being suggested that two regimes are adopted for the same investment fund.

Finally, with respect to the forecast within the sample (in-sample) using the canonic model, it is possible to observe, in Table 7, that the orders of magnitude for the forecast error, measured from the root mean square error, are relatively high even for dynamic portfolios, oscillating from 1.13% to 11.77% per month, for the 75 stock investment funds. The

exception was the fund TELE, for which the models do not seem to be capable to price adequately.

However, corroborating the evidence obtained with the significance of the traditional CAPM's Jensen's alpha, the mean squared errors are in average higher than 5% for the two last remaining columns, which are characterized by higher outperformance in relation to the Ibovespa, orders of magnitude higher than the averages obtained for the other columns in the classification of funds, which oscillate from 1.9% to 3.8%. Such results are qualitatively robust when other measurements of forecast error are used.

Comparing all the proposed frameworks, there is evidence that the model with only two regimes (one threshold) has a forecast performance slightly higher than that of the canonic CAPM, with a forecast error oscillating from 1.07% to 11.52% and an average of 3.37% against 3.41% of the canonic CAPM. Among the 75 funds analyzed, for 18 of them characterized by smaller size and lower performance, the use of the CAPM would be capable of generating better forecast results in-sample when compared to the model with one threshold.

**Table 7**  
In-sample forecast of the actual returns of the 75 stock investment funds in Brazil with the traditional CAPM  
(1998:1 - 2008:12 - 132 observations)<sup>a, b, c, d</sup>

		Overperformance (ascending order)													
Average NE (ascending order)	low	2	3	4	5	6	7	8	9	10	11	12	13	14	high
Measurement of the forecast error: Root mean square error															
<b>very small</b>	1,184%	3,008%	2,548%	3,896%	4,077%	2,536%	3,074%	2,484%	3,061%	3,478%	3,193%	4,583%	2,872%	11,772%	7,947%
<b>small</b>	4,906%	3,570%	2,320%	1,762%	4,097%	2,570%	1,749%	2,877%	3,111%	1,923%	2,106%	2,591%	3,375%	4,723%	7,850%
<b>medium</b>	5,285%	5,383%	2,275%	1,181%	1,774%	2,741%	10,247%	5,833%	2,492%	3,795%	5,955%	3,146%	4,631%	2,303%	3,501%
<b>large</b>	1,762%	2,219%	1,126%	2,590%	1,252%	1,159%	4,308%	1,631%	3,254%	2,158%	2,843%	1,779%	2,972%	2,940%	4,474%
<b>very large</b>	1,960%	1,660%	1,252%	1,777%	1,802%	2,792%	2,441%	4,438%	3,356%	5,201%	4,598%	4,017%	5,370%	4,835%	4,081%

<sup>a</sup> Panel containing all the stock investment funds in activity in Brazil and with complete time series, during the period from January 1998 to December 2008 (Source: www.fortuna.com.br)

<sup>b</sup> Average NE: arithmetic mean of the monthly time series of the net equity of each investment fund, during the period from January 1998 to December 2008, 132 observations.

<sup>c</sup> Overperformance: accumulated actual return exceeding Ibovespa (in absolute terms) of each investment fund, during the period from January 1998 to December 2008, 132 observations.

<sup>d</sup> The classification of the 75 investment funds follows Table 3.

## 6. CONCLUSIONS

The literature of asset pricing has as its mainstream the development of micro-based frameworks capable of accommodating the main stylized facts of the stocks and indices market. Even though it is an extremely legitimate concern, it may have relegated other markets to a lower level of relevance, especially the investment funds market, admittedly relevant and with one of the highest potentials for growth, as argued in Vargas and Leal (2006), according to which one of the most important questions of financial theories is related to the efficient management of portfolios.

In this context, this work adds to the discussion well developed in Matos and Rocha (2009), aiming at evidencing whether there is any common pattern or characteristic for the investment funds which cause them to be more adequately priced by a non-linear threshold

approach, having the net equity of the respective fund as decision variable. The main results suggest that the incorporation of non-linearity seems to be relevant in better accommodating the disturbance generated by the Jensen's alphas, but being capable of better pricing only a few funds with high net equity and low outperformance. The numbers also suggest there that it would be necessary to develop models in the manner of Fama and French (1993), with specific factors for investment funds, as in Matos and Silva (2010), whose results indicate that such extension is quite promising in the issue of better accommodating significant intercepts, explanatory power and, especially, with respect to in-sample and out-of-sample forecast of funds with high NE and outperformance, non-trivial conclusions, as argued in Fama and French (1993).

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<sup>i</sup> For a complete survey on the applications of the CAPM and other pricing models, see Bonomo (2004).

<sup>ii</sup> Roll, later, proved that even well diversified portfolios, such as S&P 500, could not be used as market portfolio to test the validity of the correlation beta - expected return.

<sup>iii</sup> Hansen in 1996, 1997 and 2000 worked with the inference part of such models.

<sup>iv</sup> There are over 140 funds in activity currently, but from such funds approximately half only began operating in the last ten years.

<sup>v</sup> There is vast literature on the use of proxies of the risk-free rate in Brazil. See Matos and Simonassi (2008).

<sup>vi</sup> The autocorrelation may appear for various reasons, such as inertia or slowness of the economic time series, specification trends resulting from the exclusion of important variables of the model or use of incorrect functional forms. The correlation depends on the nature of the interdependence between the error terms. But, as the error terms are not observable, it is common practice to assume that they are generated by some mechanism.

<sup>vii</sup> In other words,  $\gamma$  is restricted to a limited set  $\Gamma_n = [\underline{\gamma}, \bar{\gamma}]$ , where  $\Gamma_n = [\underline{\gamma}, \bar{\gamma}] \cap \{q, \dots, q_n\}$  and  $\{q_1, \dots, q_n\}$  is the sample of observations candidate to the TR.

<sup>viii</sup> According to Hansen (2000), a problem in the specification and in the estimation of non-linear frameworks is associated with the fact that the determination of the confidence regions depends either on large samples or on significant effects of the thresholds.