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Uncertainty and Flexibility in the Brazilian Beef Livestock Sector: the Value of the Confinement Option

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ABSTRACT

In this work, the value of existing operational flexibility is evaluated, its emergence being attributable to the Brazilian livestock alternatives for cattle fattening, i.e. by maintenance in pasture or through confinement. This crucial sector of the Brazilian economy, the second largest in the world, is highly fragmented, features low return margins and is subject to significant uncertainty factors. Confinement increases cattle fattening speed and when compared against maintenance in pasture it maximizes return on investment for farmers. In spite of this, confinement decision making is dependent upon appropriate time management. Confinement also poses risks related to the volatility of feed costs. Through the Real Options management methodology, assessment of financial growth in livestock fattening is directly linked to flexibility of timing in the transfer of cattle from pasture to confinement, with the presence of associated uncertainties. The results indicate that there is a significant increase in financial returns through containment, calculated using the return per head system. They also point to the importance of correct confinement timing to maximize returns.

Keywords: Real Options. Beef Cattle Sector. Managerial Flexibility. Financial Valuation.

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1 INTRODUCTION

According to USDA (2013) (Department of Agriculture of the United States), Brazil is the world's second largest meat producer since 2009. In 2012, domestic production was 9.2 million tons, representing approximately 16% of the total world production. In the same year, Brazil is also included as the second largest herd in the world, with 197.5 million heads, approximately 19.3 % of all world flock (USDA).

The Brazilian domestic market is among the three largest consumers, with 7.9 million tons in 2012 (USDA). According to estimates by Anualpec (2012), domestic consumption of beef per capita nationally is 34.5 kg per person per annum, similar to countries like the United States (35.3 kg) and Australia (36.1 kg). According to a survey from the Center for Advanced Studies in Applied Economics - (CEPEA / ESALQ, 2014), Gross Domestic Product (GDP) of livestock was R$332.6 billion, representing approximately 30.5 % of total GDP in agribusiness 2013, which shows that livestock has a large representation in the Brazilian economy.

In 2010, the productive line of beef turned over approximately US $ 167.5 billion (BRAZILIAN BEEF, 2011). According to Silveira (2002), the results presented in the cattle sector are due to the intense structural change in recent years occurring through various activities, i.e. the increased application of technology and more efficient management models, ensuring a prominent position on the international scene.

However the majority of producers in Brazil are small to medium sized, and the financial return of margins in the sector are small and subject to uncertainty and the market price of the final product with operating costs. An important and recurring operating decision of producers is when to confine cattle. Such decisions have a strong impact on the profitability of the business because from one perspective confinement increases fattening speed, but on the other hand it also increases production costs. Thus, determining the optimal time for confinement is of critical importance to business success.

The value of this managerial flexibility in an uncertain environment is not captured by the traditional methods of evaluating investments , such as Discounted Cash Flow (DCF) , as flexibilities requires a system with optional features. The methodology of Real Options, on the other hand, allows the assessment of managerial flexibilities to be built into the projects, processes or companies, in the presence of uncertainty. The objective of this article is to apply the methodology of Real Options to measure the value of strategic confinement timing options which are used in the beef livestock farming sector in Brazil. And so, this article seeks to highlight if this managerial flexibility (i.e. Real Options) adds, in fact, value to cattle raising, quantifying this value from the simple approach of discounted cash flow.

The beef livestock farming sector was chosen because it has elements common to other sectors of the economy, such as high levels of uncertainty in activity, the existence of managerial flexibility in (farmer) decision making, and the economic and social importance that such activity represents for the country.

This paper is organized as follows: after this introduction, we present the theoretical framework with an overview of the Brazilian livestock sector and Real Options literature reviews and modeling uncertainties for Stochastic Processes. Section 3 shows the model used. In Section 4, the modeling is demonstrated through the processing of data and the presentation of results. Section 5 presents the conclusions and suggestions for future research.

2 THEORETICAL FRAMEWORK

2.1 AGRICULTURAL SECTOR

According to the Anualpec (2012), the number of animals treated in intensive fattening feedlot via increased 67.8 % between 2003 and 2012. However, despite this growth, the number of animals treated in intensive fattening systems is still low, accounting for only about 4% of the total herd. In addition, according to the Ministry of Agriculture, Livestock and Supply (MAPA, 2012), cattle raising is present in all Brazilian states, reinforcing the economic and social importance of the activity in the country. Medeiros and Montevechi (2005) point to the existence of three stages in the production process of beef: raising, breeding and fattening, as can be seen in Table 1.

Table 1 - Phases of the Production Process of Beef Cutting

Phase	Creates	Re-creates	Fattens
Characteristics			
Animal Category at the end of phase	Calf	Lean Cattle	Fat Cattle
Average Age	Up to 8 months	From 8 to 36 months	Up to 48 months
Phase Duration	From 6 to 8 months	From 6 to 24 months	From 6 to 18 months
Weight at end of phase	From 4 to 8 bushels*	From 13 to 15 bushels*	More than 16.5 bushels*
Farming System	Extended pasture	Extensive and semi - extensive to pasture	Semi- intensive to intensive pasture and with confinement
Standard of Technology	Low and under-developed	Medium / High and Developed	Very high and developed
Farm Location	Areas distant from consumers and low fertility with land	Areas closer to consumer centers and on land with medium / high fertility	Areas close to consumer centers and in high- fertility land

*-Metric bushel, equivalent to 15kg, symbol: @
Source: Medeiros e Montevechi (2005)

According to the Agricultural Census (IBGE, 2006) , among establishments with more than 50 heads, approximately 59% of the total herd are located in specialized production units in one or two production steps, which reinforces the horizontalized structure of the process.

Silveira (2002) identifies that few producers hold the three stages of production due to cultural factors, location and land prices. In fact, specialization generates lower investment in both capital and the area of development activities (i.e. development being seen in this case as an activity in itself), while at the same time an increase in the cost of activities presents a risk to the production chain. The steps of the production process are developed properties and, according to Barbosa Andrade Souza, Grace, & Pinto (2011), can be divided into two production sub-systems: traditional subsystem (extensive) and an improved subsystem (semi-intensive or intensive). In the traditional subsystem, there is a predominance of extensive cattle with heavy reliance on obtaining nutrients through grazing, lack of investment in improvement of pastures and low annual productivity. In improved subsystems, there are increased investments in maintenance, improvement of pastures, use of mineral and nutritional supplementation, and containment systems. These subsystems can identify greater concentration of specialized properties in the stages of growing and fattening, according to Table 1.

In the stage of fattening, Barbosa (2012) reinforces the option of finishing feedlot cattle as a strategy to capture specific market conditions, increasing profitability and increasing production scale of the property. In fact, additional gains from the confinement option is possible due to the flexibility of the commercial environment in which the farmer is inserted. According IEL report, CNA and SEBRAE (2000), if the market is unfavorable at that moment, the producer has the option not to sell the animal, thus retaining the cattle on pasture or in confinement, thus reducing supply and waiting for an environment more favorable in the near future.

Thus, in properties with improved subsystem features, the farmer has the option to choose the optimal animal fattening process, pasture or confinement, and can determine when to sell the animal within the duration of the fattening stage, as shown in Table 1. Figure 1 shows the decision-making diagram for the farmer in the improved sub-system over the fattening stage.

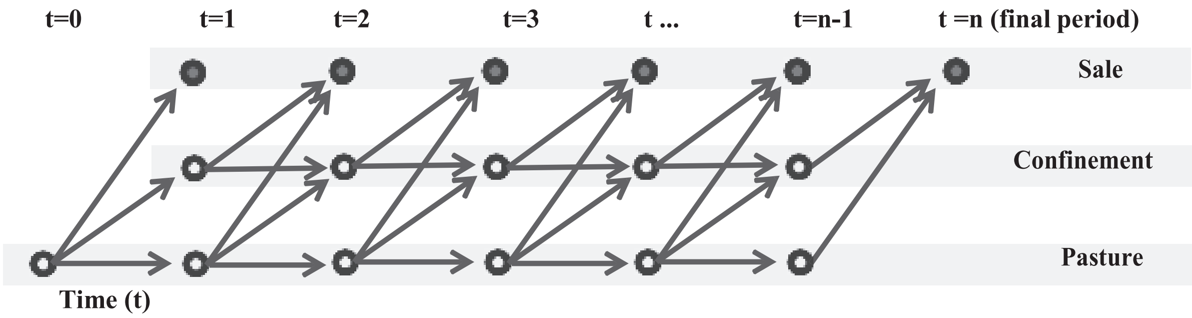

Figure 1 - Diagram of the farmer's decision-making (model) - fattening stage

The open points in Figure 1 show the possibility of the farmer's role with the closed points representing the end of flexibility. It may be noted that given the choice of confinement, the option to return the animal to pasture is extinguished because the weight gain obtained in confinement would be reduced given the energy expenditure of the animal's mobility in pasture. However the confinement strategy has restricted access because it requires high initial investment (machinery, buildings etc.) as well as additional food costs, labor, sanitation and other specific costs, further increasing the risk of the activity.

According to Lopes and Magellan (2005), confinement in power represents approximately 30.2% of the total cost of the activity (including the cost of acquiring the animal, representing 66.6 % of total costs). The main feed inputs are grain corn, soybean hulls, sunflower meal, urea and mineral supplement (GUEDES, 2011), which increases the farmer's exposure to market risks such commodities.

As seen in Maya (2003), farmer activity is subject to market uncertainty through the cost of investments and the final product, and the risks inherent in the production process such as climate, pests and genetic mutations. Thus, the skilled livestock manager, with an improved subsystem fattening stage, works with two risk factors: The first relates to the selling price of livestock cattle, and the second, confinement costs. Problems arise in assessing the economic and financial value generated by managerial flexibility due to the combination of two risk variables in the fattening stage. From the point of view of investment analysis via discounted cash flow for example, managerial flexibility of the farmer shown in Figure 1, could lead to problems in the application of traditional theories of evaluation of investment projects, i.e. because these systems of analysis fail to measure the influence of existing managerial flexibilities in the (livestock) marketplace.

2.2 REAL OPTIONS

The managerial flexibility observed in various economic sectors such as infrastructure (BRANDÃO, SARAIVA, 2008), steel industry (OZÓRIO, BASTIAN-PINTO, BAIDYA, BRANDÃO, 2013), construction (FORTUNATO, BRANDÃO, ROZEMBAUM, REBELLO, 2008) among many others, generating additional complexity in the FCD, which in turn, may not be able to adequately quantify the real value added to the project.

A pioneer, Tourinho (1979) presents methodology based on the extent of work carried out in Black and Scholes (1973) and Merton (1973) applied in the pricing of the value of options generated by managerial flexibility in the evaluation of projects using real assets (i.e. livestock). This methodology has been called Theory of Real Options (TOR), which allows for the proper assessment of the real value of real projects in uncertain scenarios.

Boyle (1988) cites the capacity of the TOR risk model to modulate two uncertainty variables using the methodology of recombinant binomial trees shown in Cox , Ross and Rubinstein (1979). Subsequently, the concept of variable bi-variate tree has been discussed in Copeland and Antikarov (2003) with two modeling uncertainty factors correlated. In Bastian-Pinto, Brandao and Hahn (2009) and Hahn and Dyer (2008) both uncertainty factors (ethanol and sugar prices) are modeled by two processes of Mean Reversion Movement (MRM) adapted to the binomial tree recombinant from the modeling of probabilities supervised and developed by Nelson and Ramaswani (1990). Thus, this approach allows for the application of the TOR methodology in real assets projects involving commodity price correlations.

2.3 CHOICE OF STOCHASTIC PROCESS

2.3.1 Project Uncertainty Factors

For this study , we will use two variable objects for pricing of the options described in Figure 2, which are: The market spot price of Livestock Cattle and confinement costs, both in Reais per Bushel (R$/@). The spot price of Livestock Cattle is the primary variable for the operating cash flow of activity, and this parameter is used to guide much of the decision-making process of the farmer (IEL; CAN; SEBRAE, 2000). The price data on the spot market of Livestock Cattle were obtained through the CEPEA site (Center for Advanced Studies in Applied Economics, the Higher School of Agriculture, CEPEA/ESALQ, 2014).

Confinement costs are the set of ingredients for the animal feed in intensive fattening stage, as an example a kernel of corn, soybean hulls, sunflower meal, urea and mineral supplement (GUEDES, 2011). For Lopes (2008), the total costs of confinement can be segregated into five main components: the acquisition of the animal, food, labor, exercise and health. For this study we will use the costs in the trough, i.e. feed costs over the other operating costs of the confinement activities. Data on the costs of confinement were obtained from estimates made by Bigma Consulting (BIGMA, 2012), industry specialists in the management of farms, project analysis and production techniques.

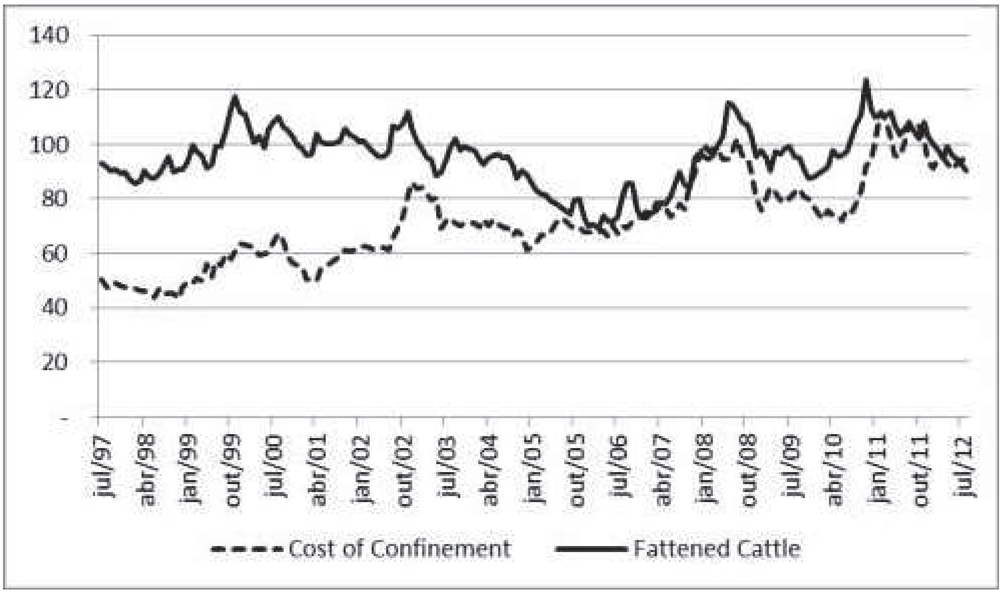

Figure 2 - Monthly average price of the deflated view (without seasonality) of Livestock Cattle and Cost of Confinement (both in R$/@)
(Translated Glossary for Chart: Custo de Confinamento = Cost of Confinement. Boi Gordo = Fattened Cattle)

2.3.2 Stochastic Processes

A common form of modeling uncertain variables in investment projects using TOR is geometric Brownian motion (GBM) (PADDOCK; SIEGEL; SMITH, 1988), in which it is assumed that the variable follows a lognormal distribution model of variance growing linearly with time. Dixit and Pindyck (1994) also show that over the short term, the spread of prices are dominated by stochastic shocks, i.e., by a geometric Brownian behavior. However empirical studies of historical data on commodity prices reveal that the average reversion models are more accurate in capturing the real behavior of these variables (HAHN; DYER, 2008).

The Mean Reversion Movement (MRM) is a Markov process in which the direction and intensity of deviation are dependent on the current price, which in turn converges to an average of market equilibrium which is assumed to be the price long-run average. The single factor Ornstein-Uhlenbeck process is the simplest form of MRM, which is defined by Equation (1):

$$dx = \eta\left(\bar{x} - x\right)dt + \sigma dz \tag{1}$$

Where x_t is the modeled variable, η the mean reversion speed, the long-term equilibrium level for which x_t converges, σ the volatility of the process and *dz* a Wiener process. The expected value and variance of the process are given by Equations (2) and (3) (PINDYCK; DIXIT, 1994):

$$E(x_t) = \bar{x} + (x_0 - \bar{x}).e^{-\eta(t-t_0)} \tag{2}$$

$$\text{var}(x_t) = \frac{\sigma_2}{2\eta}\left(1-e^{-2\eta(t-t_0)}\right) \tag{3}$$

Equations (2) and (3) show that when: t tends to infinity, then: $Var[x_t]$, tends to : $o\ ^2\!/\,2\eta$ unlike what occurs in the GBM where the variance tends to infinity as the time increases. Thus, the variance of the MRM process has a lower dispersion than the GBM model having characteristics of a non-stationary series, namely the average and variance changes with time. Thus, determining the stationarity of whether or not the series can contribute to the choice most suitable to the process for modeling the variable.

2.3.3 Selection of the Stochastic Process

One of the main verification tests of the existence of stationarity in a time series is the Dickey-Fuller test (ADF). This test unit infers the existence of root, whereby the null hypothesis (H_0) that there unit root in the series, i.e., $b = 1$, by linear least squares regression. If you can not reject the null hypothesis, we conclude that the series has a unit root and follows a random diffusion process. If the null hypothesis is rejected, there are indications that the series is a stationary process, in which the mean and variance are constant over time, featuring a mean reversion. The ADF test was applied in the representative time series of farmer uncertainty variables in the fattening stage: market prices of Livestock Cattle (CEPEA / ESALQ, 2014) and confinement costs (BIGMA, 2012), both measured in Reais per sign (R$ / @).

The EViews software was used for the augmented Dickey-Fuller test (ADF) with the deflated series in natural logarithm of Livestock Cattle Prices and Costs of Confinement. This was done using the option "intercept" in the test equation for series of Livestock Cattle prices, and the option "trend and intercept" for series of Cost of Confinement, in order to better accommodate the data series. However, for both series, the test was performed without the addition of *lags* or differentiation. The reason for this is that in studies involving derivatives (such as Real Options), the calculations of optimizations along the uncertain paths of the underlying asset, are carried out by backward induction (or from future to present). For this reason, the stochastic processes are modeled with Markov processes, in which the next step depends only on the last value of the series, regardless of the trajectory of this hitherto. For this reason, the search for stationarity should be performed on the natural logarithm of series without introduction of differentiation.

The t-statistic for the Livestock Cattle price series does not allow rejection of the null hypothesis of a unit root at a significance level of 5% (critical value: -3.425), indicating that the series presents suitability for Brownian motion. However, for variable "Confinement costs", the t-statistic shows that the series presents mean reversion behavior, as can be seen in the test results below:

- Price of Livestock Cattle - t-statistic: -2.245
- Cost of Confinement – t-statistic: -3.630

Pindyck (1999) argues that the ADF test is not sufficient to determine the choice of the stochastic process, and recommends using another approach that reflects the series behavior of the feature before price shocks, in that temporary impacts in the series show a behavior of mean reversion. To this end he suggests using the variance ratio test in equation form (4).

$$R_k = \frac{1}{k}\frac{Var\left(P_{t+k} - P_t\right)}{Var\left(P_{t+1} - P_t\right)} \quad (4)$$

The variance ratio test measures the variance of the behavior of the series as the number of lags *k* increases. In the MGB case, in which the variance grows linearly with *k* , the ratio R_k approaches 1 with *k's* growth. On the other hand, if the ratio initially increases, stabilizing thereafter at a level below 1, there is evidence of a steady process of mean reversion. The variance ratio test of the uncertainty factors of the Fattening Activity via Confinement systems was applied. The result is shown in Figure 3 and corroborates the assumption of mean reversal for both series.

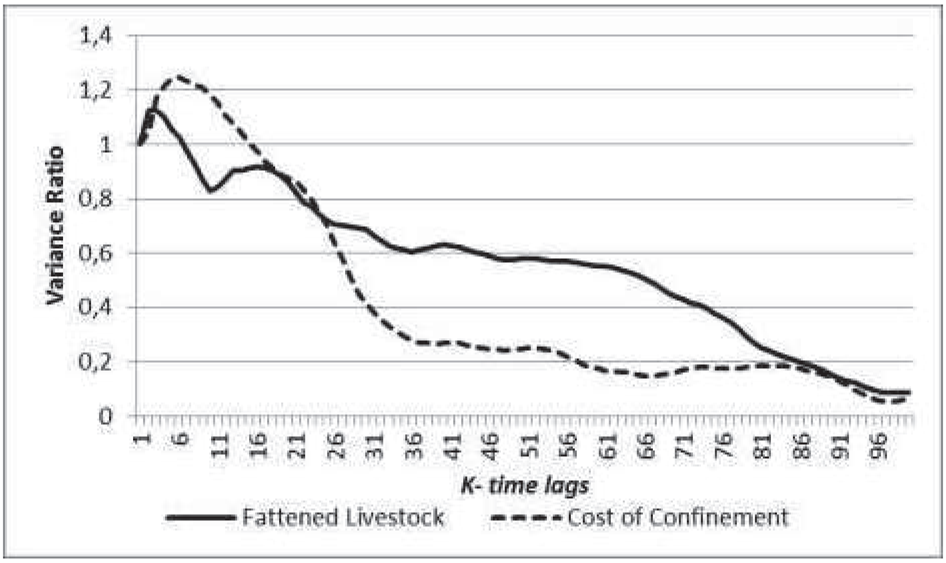

Figure 3 - Variance Ratio Test (K-defasagens tempo = K-time lags)
(Chart Translation Reference: Razao de Variancia = Variance Ratio, Boi Gordo = Fattened Livestock, Custo Confinamento = Cost of Confinement)

Overall, the tests (ADF and Variance Ratio) support the hypothesis that the factors of uncertainty pricing in Livestock Cattle and Confinement Costs can be modeled as mean reversion stochastic processes. Mean reversion processes have higher computational complexity compared to the MGB, given the greater difficulty of bringing the results to bear through the recombinant binomial tree structure.

3 METHODOLOGY

3.1 APPROACH OF RECOMBINANT BINOMIAL TREE TO THE MEAN REVERSION PROCESS

Ramaswany and Nelson (1990) propose a method that approximates the discrete continuous Ornstein-Uhlenbeck model using a recombinant binomial tree. This model uses a simple binomial sequence of length Δt with n periods, and time horizon T = $n\Delta t$. The model also uses probability as a way to manage parts of the binomial tree built by providing consistent results for the application of TOR. Equations (5) describe the values and probabilities of the binomial stochastic process to model the Ornstein-Uhlenbeck mean reversion.

$$x^{+}_{t} \equiv x_{t-1} + \sigma\sqrt{\Delta t} \equiv x_{t-1} + U \quad \text{(movement of ascent, with } U = \sigma\sqrt{\Delta t}\text{)}$$

$$x^{+}_{t} \equiv x_{t-1} - \sigma\sqrt{\Delta t} \equiv x_{t-1} + D \quad \text{(movement of descent, } D = -\sigma\sqrt{\Delta t}\text{)}$$

$$p_{t} \equiv \frac{1}{2} + \frac{1}{2}\frac{\eta\left(\bar{x} - x\right)}{\sigma}\sqrt{\Delta t} \quad \text{(probability of ascent)} \qquad (5)$$

$$1 - p_{t} \quad \text{(probability of descent)}$$

The additive binomial step (o Ornstein - Uhlenbeck process is an MRM arithmetic) is shown in Figure 4 .

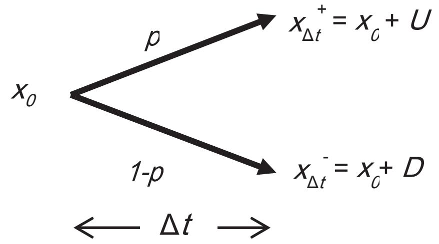

Figure 4 - Binomial additive step modeling Ornstein-Uhlenbeck

In order to demarcate, the probabilities above are within the range [1:0], Nelson and Ramaswami (1990) censor the existing probabilities that are out of range, preventing negative probabilities and / or above 100%. Equation (6) summarizes the conditions:

$$p_{x_x} = Máx\left(0, Mín\left(1, \frac{1}{2} + \frac{1}{2}\frac{\eta\left(\bar{x} - x\right)}{\sigma}\sqrt{\Delta t}\right)\right) \quad (6)$$

Since the probability of ascent and descent of each node of the tree is dependent on the recombinant x_t, level, the result converges weakly into an MRM (HAHN; DYER, 2008). However, in order to review the binomial tree with a risk-free discount rating, we need to modify the MRM for a risk neutral process. Therefore, it is necessary to penalize the long-term average by the normalized risk premium of variable x_t, represented by: λ_x/η (DIXIT; PINDYCK, 1994). Thus, Equation (7) depicts the Equation (6) adjusted for risk neutrality:

$$p_{x_x} = Máx\left(0, Mín\left(1, \frac{1}{2} + \frac{1}{2}\frac{\eta\left(\bar{x} - \lambda_x/\eta - x\right)}{\sigma}\sqrt{\Delta t}\right)\right) \quad (7)$$

3.2 BI-VARIATE MODELING WITH TWO FACTORS OF UNCERTAINTY FOLLOWING MRM

Bi-variate models allow for the possible connecting up of different stochastic processes different to the two uncertainty variables, that may or may not be correlated. The recombinant binomial lattice approach of Cox et al (1979) for two variables was initially presented by Boyle (1988). Subsequently, He (1990); Ho, Stapleton and Subrahmanyam (1995) among others, have shown the correlation of this method with models that only duplicate following GBM factors. Copeland and Antikarov (2003) present a bi-variate model, known as a quadranomial model with two correlated factors of uncertainty which both follow a GBM behavior.

Schwartz and Smith (2000) developed a mean reversion model with two factors that adopts two distinct stochastic processes to model the price of a commodity. In the article, it applied an MRM to describe the short-term behavior of this variable, and a GBM to capture the evolution of price equilibrium level over the long-term uncertainty factor. Hahn (2005) adapted the model of Schwartz and Smith (2000) with a bi-variate tree, modeling two uncertainty factors, one being built as a binomial lattice of Cox et al (1979), and the other using the Nelson and Ramaswami approach (1990), converging weakly to a mean reversion process. Bastian-Pinto, Brandão and Hahn (2009) and Hahn and Dyer (2008) apply the bi-variate tree model using two MRM processes for two project uncertainty factors using the reviewed model of Nelson and Ramaswami (1990).

In general, in bi-variate models, for each tree node there are four branches with ascent and descent probabilities associated with two variables (x and y) , as can be seen from Figure 5.

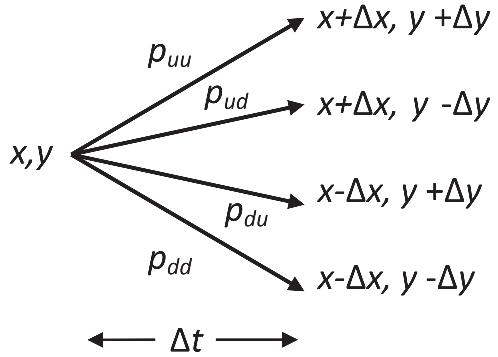

Figure 5 - Branching bivariate

Brandão and Dyer (2011) demonstrated that joint probabilities for x and y can be described as outlined in Equations (8):

$$\begin{cases} p_{uu} = \dfrac{\Delta x\Delta y + \Delta y v_x \Delta t + \Delta x v_y \Delta t + \rho_{xy}\sigma_x\sigma_y\Delta t}{4\Delta x\Delta y} \\ p_{ud} = \dfrac{\Delta x\Delta y + \Delta y v_x \Delta t - \Delta x v_y \Delta t - \rho_{xy}\sigma_x\sigma_x\Delta t}{4\Delta x\Delta y} \\ p_{du} = \dfrac{\Delta x\Delta x - \Delta y v_x \Delta t + \Delta x v_y \Delta t - \rho_{xy}\sigma_x\sigma_y\Delta t}{4\Delta x\Delta y} \\ p_{dd} = \dfrac{\Delta x\Delta y - \Delta y v_x \Delta t - \Delta x v_y \Delta t + \rho_{xy}\sigma_x\sigma_y\Delta t}{4\Delta x\Delta y} \end{cases} \quad (8)$$

Where V_x e V_y are growth rates *(drift)* of each case and ρ_{xy} the correlation between the variables. The sum of the probabilities p_{uu}, p_{ud}, p_{du}, e p_{uu} are equal to 1 (HAHN; DYER, 2008). As each node of the tree has four subdivisions, these cannot be directly censored as with the Nelson and Ramaswami methodology. Hahn (2005) circumvents this issue by applying Bayes' rule ($p(x_t \cap y_t) = p(x_t|yt)p(y_t)$) in order to decompose the joint probabilities on the product of marginal and conditional probabilities according to equations (9):

$$\begin{cases} p_u = \frac{1}{2} + \frac{1}{2}\frac{\nu_y \Delta t}{\Delta y} \\ p_d = \frac{1}{2} - \frac{1}{2}\frac{\nu_y \Delta t}{\Delta y} \end{cases} \tag{9}$$

Thus, dividing the probabilities above in Equations (8) for the corresponding equations in Equations (9) we have the following conditional probabilities for *x*:

$$\begin{cases} p_{u|u} = \frac{\Delta x\left(\Delta y + \nu_y \Delta t\right) + \Delta t\left(\Delta y \nu_x + \rho_{xy}\sigma_x\sigma_y\right)}{2\Delta y\left(\Delta x + \nu_x \Delta t\right)} \\ p_{d|u} = \frac{\Delta x\left(\Delta y - \nu_y \Delta t\right) + \Delta t\left(\Delta y \nu_x - \rho_{xy}\sigma_x\sigma_y\right)}{2\Delta y\left(\Delta x + \nu_x \Delta t\right)} \\ p_{u|d} = \frac{\Delta x\left(\Delta y + \nu_y \Delta t\right) - \Delta t\left(\Delta y \nu_x + \rho_{xy}\sigma_x\sigma_y\right)}{2\Delta y\left(\Delta x + \nu_x \Delta t\right)} \\ p_{d|d} = \frac{\Delta x\left(\Delta y - \nu_y \Delta t\right) - \Delta t\left(\Delta y \nu_x - \rho_{xy}\sigma_x\sigma_y\right)}{2\Delta y\left(\Delta x + \nu_x \Delta t\right)} \end{cases} \tag{10}$$

Therefor: $p_{u|u} + p_{d|u} = 1$ e $p_{u|d} + p_{d|d} = 1$. This formulation allows for the segregation of the four subdivisions with the joint probabilities in a sequential manner, in which the conditional probability of the variable *x* can be reviewed again using Equation (7).

Bastian-Pinto (2009) adapted to the model of Ornstein-Uhlenbeck mean reversion, those probabilities having already been formatted through the use of risk neutral Equations (11):

$$\begin{cases} p_{u|u} = \frac{1}{y}\dfrac{\sigma_x\sigma_y + \sigma\sqrt{\Delta t}\eta_x\left(\bar{x}-\lambda_x/\eta_x - x_t\right) + \sigma\sqrt{\Delta t}\eta_y\left(\bar{y}-\lambda_y/\eta_y - y_t\right) + \rho_{xy}\sigma_x\sigma}{\sigma_x\sigma_y + \sigma_y\sqrt{\Delta t}\eta_y\left(\bar{y}-\lambda_y/\eta_y - y_t\right)} \\ p_{d|u} = \frac{1}{y}\dfrac{\sigma_x\sigma_y + \sigma\sqrt{\Delta t}\eta_x\left(\bar{x}-\lambda_x/\eta - x_t\right) - \sigma\sqrt{\Delta t}\eta_y\left(\bar{y}-\lambda_y/\eta_y - y_t\right) - \rho_{xy}\sigma_x\sigma}{\sigma_x\sigma_y + \sigma_y\sqrt{\Delta t}\eta_y\left(\bar{y}-\lambda_y/\eta_y - y_t\right)} \\ p_{u|d} = \frac{1}{y}\dfrac{\sigma_x\sigma_y - \sigma\sqrt{\Delta t}\eta_x\left(\bar{x}-\lambda_x/\eta - x_t\right) + \sigma\sqrt{\Delta t}\eta_y\left(\bar{y}-\lambda_y/\eta_y - y_t\right) - \rho_{xy}\sigma_x\sigma}{\sigma_x\sigma_y + \sigma_y\sqrt{\Delta t}\eta_y\left(\bar{y}-\lambda_y/\eta_y - y\right)} \\ p_{d|d} = \frac{1}{y}\dfrac{\sigma_x\sigma_y - \sigma\sqrt{\Delta t}\eta_x\left(\bar{x}-\lambda_x/\eta - x_t\right) - \sigma\sqrt{\Delta t}\eta_y\left(\bar{y}-\lambda_y/\eta_y - y_t\right) + \rho_{xy}\sigma_x\sigma}{\sigma_x\sigma_y + \sigma_y\sqrt{\Delta t}\eta_y\left(\bar{y}-\lambda_y/\eta_y - y_t\right)} \end{cases} \quad (11)$$

Again, these can be directly censored if their values are above 1 or below 0. The marginal probabilities of ascent and descent movements fot the process y_t are described by Equation (12):

$$\begin{cases} p_u = \frac{1}{2} + \frac{1}{2}\dfrac{\eta_y\left(\bar{y}-\lambda_y/\eta_y - y_t\right)\sqrt{\Delta t}}{\sigma} \\ p_d = \frac{1}{2} - \frac{1}{2}\dfrac{\eta_y\left(\bar{y}-\lambda_y/\eta_y - y_t\right)\sqrt{\Delta t}}{\sigma^y} \end{cases} \quad (12)$$

In the model adapted by Hahn and Dyer (2008), the marginal probabilities of both variables may be reviewed to reflect the modeling of two MRM processes, as shown in Figure 6.

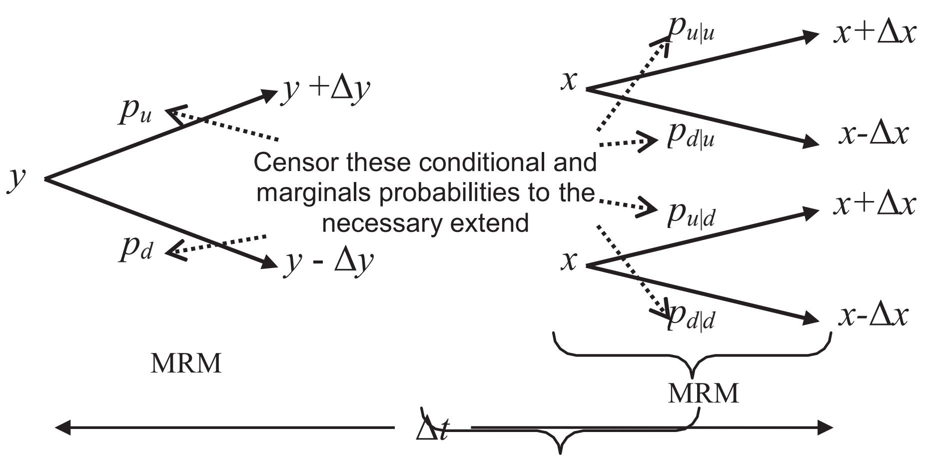

Figure 6 - Node bi-variate Censored in two MRM processes

4 MODELLING AND RESULTS

4.1 ESTIMATION OF THE MAIN PARAMETERS EMPLOYED

For the price of Livestock Cattle the Livestock Cattle Price Indices ESALQ / BM & FBOVESPA were used, said indices are available online at CEPEA (CEPEA/ESALQ, 2014).

Data from "cost confinement" was obtained through Bigma Consulting (2012). Both data were measured in Brazilian Reais per bushel (R$/@) for the period July, 1997 to August, 2012.

The series dealt with on a monthly basis were deflated by the IGP -M (General Index of Prices internal-availability, source: the Getúlio Vargas Foundation). Additionally, the effects of seasonality inherent to the dynamics of the annual cycle, such as the dry season, were purged.

The mean reversion variable parameters were obtained through first order autoregressive process AR (1) on the logarithm of the deflated variables. The results are shown in Table 2.

Table 2 - Parameters for Modeling by MRM

Parameters	η_{aa}	σ_{aa}	$\bar{x}$* (R$/@)	x_0 (R$/@)
Fattened Cattle	0.68	13.60%	94.08	90.48
Cost of Confinement	0.22	15.21%	85.33	71.95

The values of $\bar{x}$ for both stochastic variables modeled in Table 2 already contain their values with adjustments for risk neutrality (therefore: $\bar{x}^* = \bar{x} - \lambda_x / \eta_x$).

The interdependence between the uncertainty factors was modeled using the serial correlation of data, with the value of ρ_{xy} = 15.33% . 7 % p.a. was used as the risk free rate .

4.2 FARMER CASH FLOW AND REAL OPTIONS EVALUATION

Modeling of options for phase "fattening" for the production process of Cattle Fattening was restricted, as shown in Table 1. This period has as its ultimate goal the increase of the animal's weight, increasing thus the economic return of the property.

According to Medeiros and Montevechi (2005), the fattening period lasts from six to eighteen months, with the weight of the animal up to the end of the process topping sixteen and a half bushels (16.5@). During the eighteen months of the fattening process, producers may choose to fatten animals via pasture, or use intensive fattening technique by means of confinement structures, to further increase the animal's weight. In the eighteenth month, fattening options expire and the sale of the animal is mandatory, characterizing the end of the fattening period.

Based on the parameters shown in Table 2, the following uncertainty factors were estimated over a project period of eighteen months: Price of Livestock Cattle and Cost of

Confinement. Each combination of these stochastic variables allowed us to estimate cash flows for the farmer. Given the heterogeneous structure of existing properties in the country, using the unit operating cash flow, or per animal, in order to more broadly capture the value added by the strategic confinement option. The option value is found in the differential cash flow, i.e. comparing the producer of cash flow with the situation in which there is only an extensive fattening option (the pasture) with the other cash flows including intensive fattening options in strategic confinement.

In Table 3, the structure of estimated cash flows is shown as follows: the spot price is represented by the variable price of Livestock Cattle at market value in R$ / @ ; Weight is the cumulative weight gain nutrition via grazing in @; Δ Confinement Weight is the accumulated variation of weight gained using the confinement system measured in @ ; Purchase Price is the animal's acquisition costs equivalent to the selling price of Lean Cattle; Operating costs refer to costs on a farm in R$/animal; and Cost of Confinement is measured in R$/@ .

Table 3 - Estimated Cash Flow for Fattening on Pasture and Confinement

Pasture Flow	Confinement Flow
Spot Price x Weight	**Spot Price x Weight**
	(+) Δ Confinement Weight
(-) Purchase Price	(-) Purchase Price
(-) Operating Costs	(-) Operating Costs
	(-) Confinement Costs x Δ Confinement Weight
(=) Pasture Flow	**(=) Confinement Flow**

The purchase price reflects the initial investment in the acquisition of the animal before the weight characteristics and age of slaughter. In this study, we used the unfattened cattle market prices to reflect the opportunity cost of introducing the animal to the fattening stage, turning it into Livestock Cattle, or bringing the animal to market.

The nationwide operating costs in R$/hectare/p.a. were provided by CEPEA (CEPEA/ESALQ, 2014). Costs serve as a database to monitor the input price index used in the production of Brazilian livestock, which was also prepared by CEPEA (2014) . The above values have been adapted to reflect the unit costs per animal based on the ratio of 1.08 animals per hectare, according to IBGE (2006).

- Purchase Price (R$/animal): 1.170.00
- Effective Operating Cost (EOC) (R$/animal/p.a.): 303.65

The acquisition and operating costs are grow at the risk-free rate to reflect the cost of capital invested over the duration over the option exercise period. In general, the expected weight gain with the confinement has been increasing over time, due to improvements in nutritional techniques, better planning and investments. Corrêa (1996, p. 61) estimated that the average daily gain (ADG) should be between 0.7 kg to 1.2 kg, however Nogueira and Coan (2011) portray that feedlots work with planned rations for ADG from 1.7kg to 1.8 kg.

According to Barbosa (2011), in improved subsystems, where there is greater investment in diets and supplements, the average ADG lies between 0.6 kg and 0.8 kg in the rainy season and 0.5 kg and 1.0 kg in the dry season. ADG values in confinement 1.7kg and 0.7 kg for nutrition on pasture were used in the modeling.

4.3 RESULTS AND SENSITIVITY

Using the methodology described above, it was calculated that the value of R$ 86.51 per animal is the equivalent value of the Real Option of strategic confinement. This amount represents 7.4% of the value used for Lean Cattle in the baseline scenario.

As beef cattle activity is present in all states of Brazil, we identified the need to sensitize the main assumptions used in the model, in order to verify the elasticity of the value of the confinement option. Figure 7 shows the value of the strategic confinement option expressing the principal parameters described above. The featured data shows the option's value to the basic parameters used.

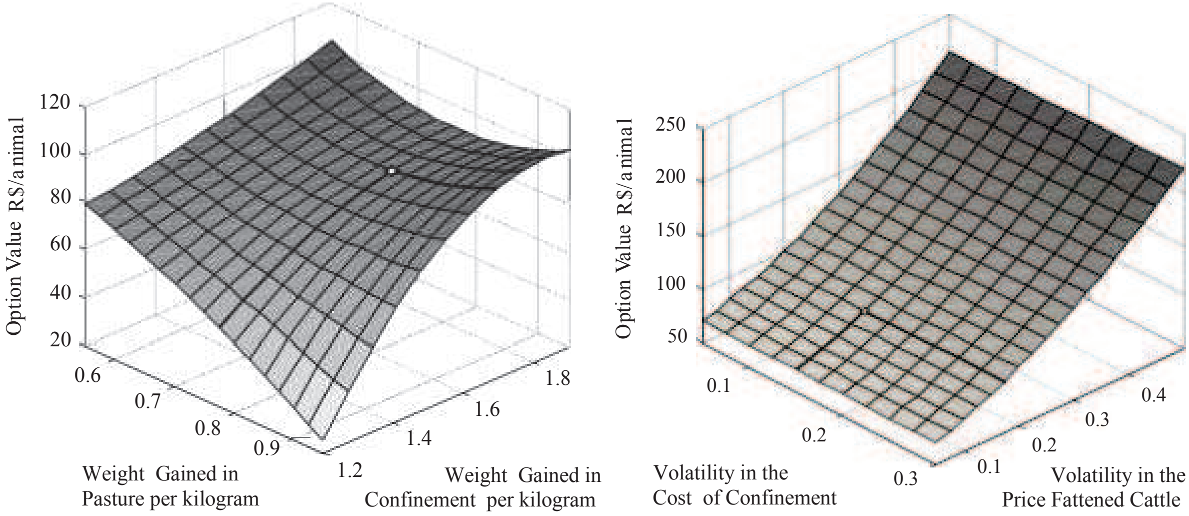

Figure 7 - Sensitivity of the Value of Strategic Confinement Options as to GPD and volatility. (Translated Glossary of chart terms: GDP Pasto Kg = Weight Gained in Pasture per kilogram, GDP Confinamento Kg = Weight Gained in Confinement per kilogram, Valor Opcao = Option Value, Vol. Custo Confinamento = Volatility in the Cost of Confinement, Vol. Preco Boi Gordo = Volatility in the Price Fattened Cattle)

We can point out the high amplitude of the value of strategic confinement to sensitizing to operating parameters: Weight gain in both the pasture and in the confinement. Moreover, it can be attributed to the high sensitivity of optional values to the volatility of Livestock Cattle price, and in conversely the low sensitivity to volatility in Cost of Confinement.

For comparison, the series were also modeled using the quadranomial model suggested in Antikarov and Copeland (2003) in which the two factors of uncertainty are modeled through a geometric Brownian diffusion process. Table 4 depicts the value of the confinement strategic options comparing the results of the two stochastic processes . As can be seen, there is a difference of 29% in the results between the two processes, which further increases the importance of defining stochastic processes to be used in modeling by Real Option Theory.

Table 4 - Comparison of Results Between MRM and GBM Processes

Stochastic Process	**MRM**	**GBM**	**Δ%**
Value of the option of strategic confinement (R$/animal)	86.51	111.29	29%

5 FINDINGS & RECOMMENDATIONS

In this article theoretical tools are described for using the Real Options Theory (TOR) applied to a sector with high uncertainty and high capacity of interference on the part of the decision maker: the agricultural sector of beef livestock.

The horizontalized structure of livestock in the country intensifies the market risk of main inputs and final products, here portrayed by the uncertain variables of sale price and cost of feed. At the same time, it does allow the producer to hold margin in selecting the best nutrition system, for example, feeding or pasture via the confinement systems, as well as the animal selling being delayed in adverse market conditions. The TOR approach was used to price the value the confinement decision strategy in this environment.

The study results confirm that TOR can capture the financial value of the strategic confinement flexibility of farmers. In addition, it is evident that managerial flexibility is a powerful tool both to maximize value as a protection against exogenous uncertainty for the farmer (as, say, a *hedge*).

However it is also clear that the correct exercise of these flexibilities, namely: the timing of confinement exercises, compared to the behavior of mapped uncertainties, it is of fundamental importance to this maximization of value. Remember that the managerial flexibility confinement time is in fact exercised by the farmer, but intuitively and without some form of proper measurement of the financial value arising from this. We can also highlight the high sensitivity of the value of Real Option to the volatility of the price of Livestock Cattle, which is the main risk factor of the farmer.

The methodology of this Article extends to the actual measurement capacity value of livestock activity which, can be applied in risk management studies of the activity, as well as in the assessment of activity in specialized properties in other stages of production of meat, such as breeding and reproduction.

Other sectors of the economy and agriculture can also benefit from this methodology, measuring the value of managerial flexibility already known such as the possibility of production changes between ethanol and sugar, from the same source input: sugarcane, among others.

REFERENCES

ANUALPEC. **Anuário da pecuária brasileira**. São Paulo: FNP Consultoria & Comércio, 2000.

BARBOSA, F. A. **Confinamento**: planejamento e análise econômica. Disponível em: <http://www.agronomia.com.br/conteudo/artigos/artigos_confinamento_analise_economica.htm>. acesso em: 25 set. 2012.

BARBOSA, F. A. et al.Dietas de alto concentrado para terminação de bovinos de corte. In: ENCONTRO DOS MÉDICOS VETERINÁRIOS E ZOOTECNISTAS DOS VALES DO MUCURI, JEQUITINHONHA E RIO DOCE, 32., 2011, Teófilo Otoni (MG). **Anais...** Teófilo Otoni: SRMVVM, 2011.

BASTIAN-PINTO, C. L.. **Modelagem de opções reais com processos de reversão à média em tempo discreto**: uma aplicação na indústria brasileira de etanol. 2009. Tese (Doutorado) - Pontifícia Universidade Católica Rio de Janeiro (PUC-RJ), Rio de Janeiro, 2009.

BASTIAN-PINTO, C.; BRANDÃO, L.; HAHN, W. J. Flexibility as a source of value in the production of alternative fuels: the ethanol case. **Energy Economics**, v. 31, n. 3, p. 411-422, 2009.

BIGMA Consultoria. Disponível em: <http://www.bigma.com.br>. Acesso em: 28 nov. 2012.

BLACK, F.; SCHOLES, M. The pricing of options and corporate liabilities. **The Journal of Political Economy**, v. 81, n. 3, p. 637-654, 1973.

BOYLE, P. A. A lattice framework for option pricing with two state variables. **Journal of Financial and Quantitative Analysis**, v. 23, n. 1, p. 1-12, 1988.

BRANDÃO, L. E. T.; SARAIVA, E. C. G. The option value of government guarantees in infrastructure projects. **Construction Management and Economics**, v. 26, n. 11, p. 1171–1180, 2008.

BRANDÃO, L. E. T.; DYER, J. S. Valuing real options projects with correlated uncertainties. **Journal of Real Options**, v. 1, p. 18-32, 2011.

BUENO, R. L. S. **Econometria de séries temporais**. São Paulo: Cengage Learning. 2011.

CEPEA/ESALQ. Centro de Estudos Avançados em Economia Aplicada. Disponível em: <http://www.cepea.esalq.usp.br>. Acesso em: 15 jun. 2014.

COPELAND, T. E.; ANTIKAROV, V. **Real options**: a practitioner's guide. New York: Texere, 2003.

CORREA, A.N.S. **Gado de corte**: o produto pergunta, a Embrapa responde. Brasilia: EMBRAPA-SPI, 1996. 208p.

COX, J. C.; ROSS, S. A.; RUBINSTEIN, M. Option pricing: a simplified approach. **Journal of financial Economics**, v.7, n. 3, p.229-263, 1979.

DIXIT, A. K.; PINDYCK, R. S. **Investment under uncertainty**. Princeton: Princeton University Press, 1994.

FORTUNATO, G. et al. Valor da opção de abandono em lançamentos imobiliários residenciais. **RAC-Eletrônica**, v. 2, n. 3, p. 531-545, 2008.

GUEDES, J. F. B. **Avaliação de diferentes custos de alimentação para bovinos nelore terminados em sistemas de confinamento e semi-confinamento**. 2011. 35 f. Monografia (Graduação em Engenharia Agrônoma) – Faculdade de Agronomia e Medicina Veterinária da Universidade de Brasília (UnB), Brasília (DF), 2011.

HAHN, W. J. **A discrete-time approach for valuing real options with underlying mean-reverting stochastic processes**. 2005. PhD Dissertation, The University of Texas, Austin (TEX), 2005.

HAHN, W. J.; DYER, J. S. Discrete time modeling of mean-reverting stochastic processes for real option valuation. **European Journal of Operational Research**, v. 184, n. 2, p. 534-548, 2008.

HE, H. Convergence from discrete to continuous-time contingent claims prices. **Review of Financial Studies**, v. 3, n. 4, p. 523-546, 1990.

HO, T. S.; STAPLETON, R.; SUBRAHMANYAM, M. Multivariate binomial approximations for asset prices with nonstationary variance and covariance characteristics. **Review of Financial Studies**, v. 8, n. 4, p. 1125-1152, 1995.

IBGE. Fundação Instituto Brasileiro de Geografia e Estatística. **Censo Agropecuário 2006**: agricultura familiar, Brasil, grandes regiões e unidades da federação. Rio de Janeiro, RJ: IBGE, 2006.

IEL; CNA; SEBRAE. **Estudo sobre a eficiência econômica e competitiva da cadeia agroindustrial da pecuária de corte no Brasil**. Brasília: IEL, 2000. p. 248.

LOPES, M. A.; MAGALHÃES, G. P. Análise da rentabilidade da terminação de bovinos de corte em condições de confinamento: um estudo de caso. **Arquivo Brasileiro Medicina Veterinária e Zootecnia**, v. 57, n. 3, p. 374-379, 2005.

MAPA. Ministério da Agricultura, Pecuária e Abastecimento. Disponível em: <http://www.agricultura.gov.br/animal/especies/bovinos-e-bubalinos>. Acesso em: 20 set. 2012.

MAYA, F. L. A. **Produtividade e viabilidade econômica da recria e engorda de bovinos em pastagens adubadas intensivamente com e sem o uso da irrigação**. 2003. Dissertação (Mestrado) - Escola Superior de Agricultura Luiz de Queiroz, Piracicaba (SP), 2003.

MEDEIROS, A. L.; MONTEVECHI, J. A. B. Modelagem da equação de previsão do preço da arroba de boi gordo através da regressão linear múltipla. In: SIMPEP, 12., 2005, Bauru (SP). **Anais…** Bauru: SIMPEP, 2005.

MERTON, R. C. Theory of rational option pricing. **The Bell Journal of Economics and Management Science**, v. 4, n. 1, p. 141-183, 1973.

NELSON, D. B.; RAMASWAMY, K. Simple binomial processes as diffusion approximations in financial models. **Review of Financial Studies**, v. 3, n. 3, p. 393-430, 1990.

NOGUEIRA, M. P.; COAN, R. M. Avaliação e decisão no confinamento. In: ENCONTRO DE CONFINAMENTO: GESTÃO TÉCNICA E ECONÔMICA, 6., 2011, Jaboticabal (SP). **Anais...** Jaboticabal, 2011. p. 25-56.

OZORIO, L. M. et al. Investment decision in integrated steel plants under uncertainty. **International Review of Financial Analysis**, v. 27, p. 55-64, 2013.

PADDOCK, J. L.; SIEGEL, D. R.; SMITH, J. L. Option valuation of claims on real assets: the case of offshore petroleum leases. **Quarterly Journal of Economics**, v. 103, n. 3, p. 479-508, 1988.

PINDYCK, R. S. The long-run evolution of energy prices. **Energy Journal**, v. 20, n. 2, p. 1-27, 1999.

SCHWARTZ, E. S.; SMITH, J. E. Short-term variations and long-term dynamics in commodity prices. **Management Science**, v. 46, n. 7, p. 893-911, 2000.

SILVEIRA, R. L. F. **Análise das operações de Cross Hedge do bezerro e do Hegde do boi gordo no mercado futuro da BM&F**. 2002. Dissertação (Mestrado) - Escola Superior de Agricultura Luiz de Queiroz, Piracicaba (SP), 2002.

TOURINHO, O. A. F. **The valuation of reserves of natural resources: an option pricing approach**. 1979. PhD Dissertation (Doctoral) - University of California, Berkeley (CA), 1979.

USDA. United States Departament of Agriculture. Disponível em: <http://www.fas.usda.gov/psdonline/>. Acesso em: 15 fev. 2013.